The University of Alberta

A UNIVERSITY ENROLMENT PROJECTION MODEL

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled A UNIVERSITY ENROLMENT PROJECTION MODEL submitted by ROBERT ALAN McLEAN in partial fulfilment of the requirements for the degree of MASTER OF BUSINESS ADMINISTRATION.

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ABSTRACT

This paper summarizes the four methods generally used for projecting school and university enrolments and describes a specific application of two of them, the Cohort Survival Method and the Transition Coefficient Technique. It is shown that the former is a special case of the latter, and a general mathematical model is described which can be used for both. An exponential smoothing method is used for projection of the transition coefficients, which provides for the continual revision of the projections as new data become available without requiring a major technical restructuring of the model.

The results of implementation of the model in the projection of high school graduates are given, and the relationship to the university enrolment model is shown.

ACKNOWLEDGEMENT

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Introduction

This paper describes a model which is presently being applied at the University of Alberta for the purpose of projecting potential enrolments over the planning horizon of the university.

There is little about the structure of this model that has not been described previously by other authors in terms of general enrolment projection models. It is not the intent of this paper to provide an exhaustive analysis of the relative merits of various methods of enrolment projection or to develop new or more sophisticated versions. Rather, it is intended to summarize what work has been done and to describe the application of this work to a specific situation.

The development of mathematical models, which allow a concise expression of the complex interrelationships present in an educational institution, has provided a powerful tool for the measurement of the effects of alternative actions facing decision makers. All too often, however, the potential power of these tools is not realized in practice. This may occur for several reasons: a lack of understanding of the techniques involved, an unwillingness to apply the methods to a process which many consider unquantifiable, or the cost of implementation exceeding the perceived benefits to be derived. More often, the models cannot be used because the data required for their implementation is not available, or the overall organizational structure, policy and process characteristics are not adequately defined. This is particularly true of the comprehensive, "total system", models presently in vogue.

The choice of model, therefore, is contingent upon the informational needs it is expected to meet, the information resources which are available for input, and the resources required for the task of implementation. This task is as crucial as the initial model development and is likely to be at least as formidable.

CHAPTER I

Methodology

Philosophy of Admissions

There are two fundamental philosophies of admission of students into the education system. The first is that the educational system is the means by which the human resources of a nation are developed; and, therefore, the size and distribution of education enrolments is determined by the future needs of the nation and is controlled by the state. This philosophy is practised in some parts of Europe and in many developing countries where the deficit in human resources is generally more severe than in the developed nations. The enrolment projection models used under this philosophy are economic and sociological in nature and designed to maximize the growth of economic well-being of the nation without particular regard for the demand for education by the members of society at large. Methods and models are described by Davis, Liu, and Barnard.[1]

The second philosophy is more applicable to the present situation in North America. It considers the opportunity for education to be the right of every citizen, and the provision of the opportunity to be the duty of the state. Under this philosophy, the

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[1]Russell C. Davis, Planning Human Resource Development, (Chicago: Rand McNally, 1966); Bangee Alfred Liu, Estimating Future School Enrolment in Developing Countries, (UNESCO), Population Studies, No. 40); Jean Barnard, "General Optimization Model for the Economy and Education" in Mathematical Models in Educational Planning, (Paris: Organization for Economic Cooperation and Development, 1967).

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projection of educational enrolments becomes the projection of age specific population growth combined with forecasting the demand for education, which arises from the various sectors of the population. Four methods of projection have been used, characterized by differing degrees of reliance upon cause-effect relationships, and consequently, requiring varying levels of detail in the input data. These methods are: Curve Fitting to historical enrolment data, Demographic Ratio analysis, Cohort Survival analysis, and Transition Coefficient analysis.

Methods

Curve Fitting

"Enrollment projections by the curve-fitting method consist of determining the functional relationship which exists between past enrollments and years."[2] The only input data requirements are the historical time series of enrolments for the educational sector being analyzed. The underlying assumptions, however, are extremely restrictive. These are: that past enrolment trends will continue, and that new factors will not be introduced in the future. This method is generally only useful for short-term projection in highly stable environments with simple cause-effect relationships. It might be used, for example, to project the number of new pupils entering grade I, where the net population growth of six-year-olds has been stable and is expected to continue to be the same, and where

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[2] L. J. Lins, Methodology of Enrollment Projections for Colleges and Universities, (American Association of Collegiate Registrars and Admissions Officers, 1960), p. 8.

all children must enter grade I at age six. It may not be useful in the same situation, however, for projecting the total grade I population, which is determined by student ability and promotional practice, in addition to the rate of new pupil growth. The details of the methods of curve fitting are not pertinent to this discussion and may be found in most general statistics texts. Boling and Gardiner[3] also provide a good analysis of the various methods.

Demographic Ratio Analysis

This method uses historical enrolment and demographic data to determine the relationship which exists between specific age group populations and enrolments in the educational system, and applies the projected relationship to demographic forecasts to develop future enrolments. In this case, both historical enrolments and census data must be known, and projections of future populations must be available. Since the short-term stability of the population is greater than that of school enrolments, the underlying assumptions are somewhat less critical than in the curve-fitting methods, but the traditional assumptions of demographic projection still apply; and, further, it must be assumed that the historical relationship is representative of the relationship which will hold in the future.

This method of projection is probably the one most widely used today, particularly for the projection of national and regional

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[3] Edward J. Boling and Donald A. Gardiner, Forecasting University Enrollment, (Knoxville: The University of Tennessee, 1952).

enrolments. Certainly for long-range projections, particularly of elementary school enrolments, it provides the best available estimate. For the specific institution of higher education, however, it is not an effective tool for operational planning as the results are usually aggregated by region and by total institutional enrolment, rather than by specific program level. The results are extremely valuable as an independent verification of the aggregated results of more detailed analysis. Of particular interest in this regard are Hanson's projections of grade XII enrolment in college areas in Alberta,[4] and Hassbring's analysis of enrolment for the Department of Sociology at the University of Alberta.[5]

The latter reference also provides a more detailed description of this method and a good indication of the types of assumptions which must be made.

Cohort Survival

The Cohort Survival method uses a succession of ratios to determine the extent to which the members of a given grade will survive to a higher grade at some later period in time. While the concept of survival may appear relatively simple at first glance, the underlying

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[4] E. J. Hanson, Population Analysis and Projections, College Areas in Alberta, (Edmonton: Department of Education, Provincial Board of Post-Secondary Education, 1968).

[5] Lars Hassbring, "Enrollment, Staff, and Space Projections", unpublished report of the Population Research Laboratory, University of Alberta.

factors can be complex. In general, the enrolment in any stage (grade) is made up of some members of the immediately lower stage in the previous year; some members of the same stage in the previous year, who failed to progress to the following stage; and some new entrants from outside the system. Thus, the survival ratio used to advance a cohort from one stage to the next is a single number approximation of a multivariate relationship. However, if it is assumed that the historical relationships between the underlying variables will remain unchanged in the future, reasonable estimates of future enrolments can be derived. This method of forecasting is limited to a period equal to the normal duration of student progress from the start to the end of the segment being analyzed. That is, the most recent historical data for the lowest stage of the segment provides the base for projecting the enrolment in the final stage at some later period. For example, the enrolment in grade I this year will provide the base for the grade XII enrolment in eleven years; and, therefore, the base for the number of high school graduates in twelve years.

The main benefit of this method is derived from the fact that historical data on school enrolments are readily available and are more quickly updated than the demographic data required for the previous method. Further, projections of the numbers of students who will enter the post-secondary segment of the education system can be made for twelve years in the future, based upon the enrolment which is presently in the primary- and secondary-school segments. The method is limited by the stability of the historical survival ratios for each stage in the

process, and the accuracy with which the future survival ratios can be estimated. Various regression and time-series analysis techniques are available for this purpose, and the most suitable method for the specific situation is determined by the characteristics of the application being considered. A more sophisticated approach would be to determine the values of the underlying variables, i.e., net migration, student ability and promotional practice, and forecast changes in survival ratios as a function of changes in these values. This is often not possible within the economic and data availability constraints imposed upon the analysis, and the more naive techniques must suffice.

Transition Coefficient Method

The Demographic Ratio Analysis and the Cohort Suvival techniques are special cases of the general Transition Coefficient method. In the former, the contributing states are the defined population categories, and the accepting states are the education system segments. The ratios which are determined are the transition coefficients between these states.

Similarly, the cohort survival technique provides a set of transition coefficients from the immediately lower grade (state) to the grade in question.

In the general case, the number of possible states is theoretically unlimited. In practice, it is limited by the availability of data, the computational power required to use the model, and the application of the particular analysis. Regardless of the level at

which states are defined, if a classification scheme is given, which includes all possible states within the educational system; students' flow through the system can be defined by a matrix of transition coefficients which represent the probabilities of moving from any given state to any state in the following period. Hence, if the vector of populations of all states at time t is known, the vector at time t + 1 can be determined by matrix multiplication of the transition matrix and the population vector. This process assumes, however, that no one enters or leaves the system. The latter can be corrected by including a set of absorbing states in the classification scheme, which represent final graduation at different levels of attainment. New entrants to the system can be introduced by the addition of a new entrant vector to the population vector at each time period. Excellent descriptions of this method are given by Thonstad[6] and Baisuch and Wallace.[7] This model provides a more formidible tool than the three methods previously described, for the analysis of the effects of change in input variables, or parameters, upon the students' flow through the system and upon the future populations of each of the system's states. Input data must be provided in greater detail than in the previous models; and the problem of projecting the number of new entrants to the system in each period and the behaviour of the transition

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[6]Tore Thonstad, "A Mathematical Model of the Norwegian Educational System: in Mathematical Models for Educational Planning, ibid.

[7]Allen Baisuch and William A. Wallace, "A Computer Simulation Approach to Enrollment Projection in Higher Education", Socioeconomic Planning Sciences, Vol. VI (1970), pp. 365-81.

coefficients over time, is at least as hazardous as the three previous methods. The transition coefficients represent the net result of educational policy, student ability, socioeconomic pressures, and, to some extent, the vagaries of student selections. Ideally, by increasing the number of permissible states, variations in these parameters could be measured and included in the model. However, the practical limitations on processing large arrays, prohibit the amount of subdivision required, and the same old assumptions concerning the relationship of underlying variables must be made.

Summary

The general methodology of projecting student enrolments has been well developed and documented and offers a wide choice of alternatives for institutional use. The implementation of these methods to meet specific needs is not a simple task and requires major tradeoffs between the level of details required, the quality of the input data available, and the cost of implementation of the projection system.

CHAPTER II

The Enrolment Model

The enrolment model used in this application is made up of two parts: one to forecast the numbers of high school graduates in the province that achieve Senior Matriculation standing, and the second to provide the specific university enrolments. The first part is necessary in order to provide the projected populations from which the majority of the new entrants to the university are derived.

The High School Graduate Model

Background

For the purpose of developing the population of the university new entrant group, we must be able to project the size of the major source of these students. Previous analysis has indicated that approximately 95 percent of this group comes from high school graduates within the province who have achieved Senior Matriculation status. This historical growth in numbers of these graduates is shown in Table I and Figure I.

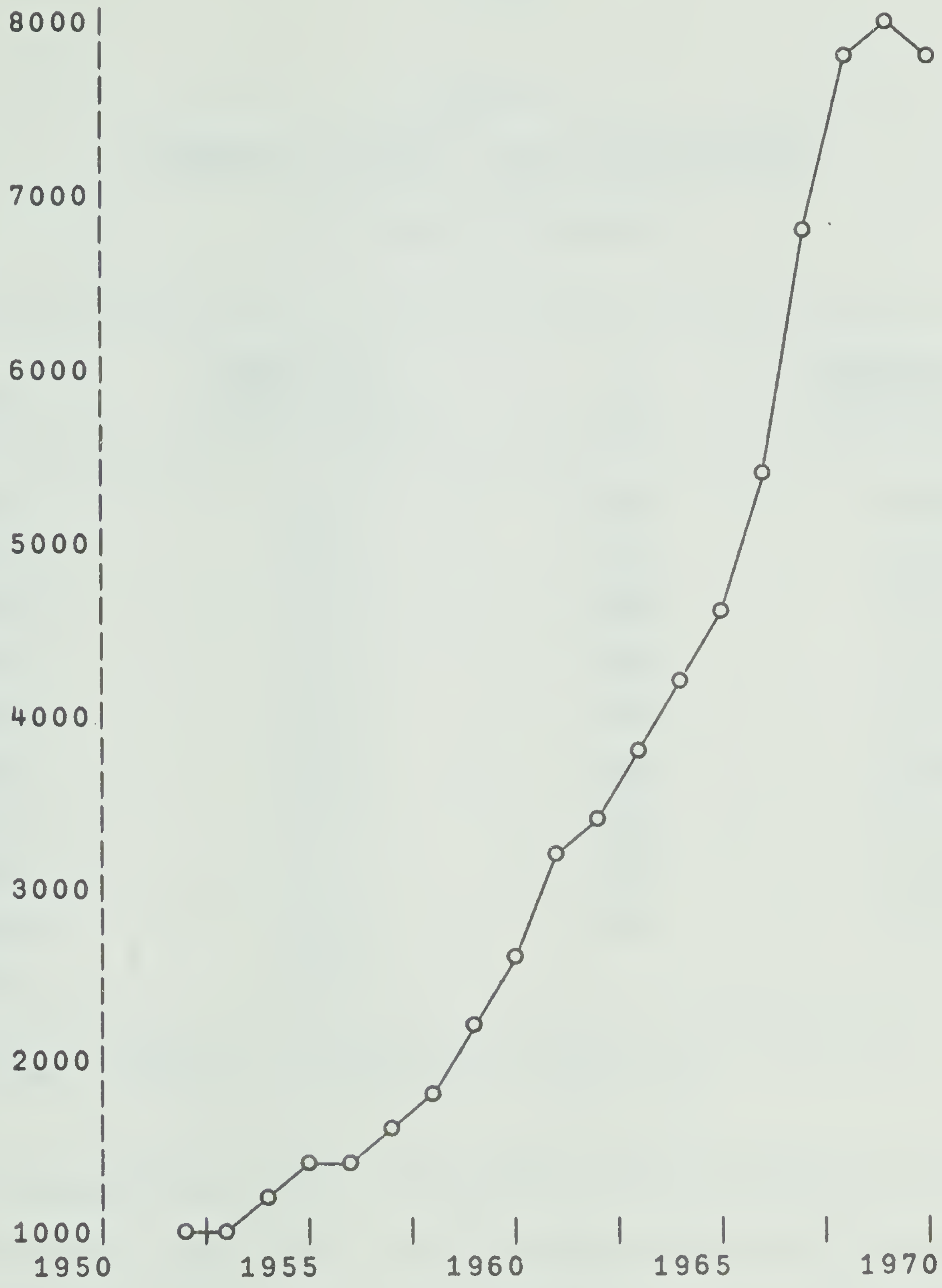

FIGURE 1

Growth of Senior Matriculant Numbers

Table I

Numbers of Alberta Senior Matriculants

By Year of Graduation

Year	Number of Matriculants	Year	Number of Matriculants
1952	1,024	1962	3,492
1953	1,098	1963	3,710
1954	1,237	1964	4,213
1955	1,426	1965	4,588
1956	1,458	1966	5,392
1957	1,668	1967	6,886
1958	1,874	1968	7,819
1959	2,121	1969	7,910
1960	2,502	1970	7,787
1961	3,202		

Source: Alberta: Department of Education.

For the period to 1968, the growth appears regular and best approximated by a power curve. Such an approximation has obvious long-run projection errors in view of limitations which are imposed by the foreseeable overall population growth. Further, the observations for 1969 and 1970 differ markedly from such an approximation.

Visual analysis of the growth curves of each grade in the school system indicated that there were identifiable growth patterns which could be followed through the system. (See Appendix A for enrolment statistics and growth curves.) The most significant of these patterns

was the major enrolment increase that occurred in grade I over the two year period from 1952 to 1954. This increase can be followed through to grade XII where it appears from 1963 to 1966. The increase in spread to three years can be accounted for by the fact that grade XII can be taken in either one or two years. The somewhat greater growth in Matriculants in this period compared to the previous period can be explained by this enrolment increase. However, the major increase in Matriculants occurred in the period from 1966 to 1968, a period when the enrolment growth in grade XII was relatively low.

Again, visual analysis of the ratios of Senior Matriculants to students enrolled in grade XII, showed a major discontinuity in this period. Figure 2 shows that this ratio increased from approximately 25 percent in 1966 to approximately 35 percent in 1968. Subsequent conversation with the Department of Education indicated that this shift occurred at a time when promotional practice was changed, and that such a shift was an anticipated effect of the change.

As a result of these investigations, it was felt that the Cohort Survival Method could be used to provide realistic projections of Senior Matriculants. In order for such a method to be fully effective for the purpose intended, i.e., an administrative tool for testing policy alternatives, the model must have two features. Firstly, it must be capable of providing a realistic estimate of the future survival ratios, using the most current data available without requiring a major technical reevaluation. Secondly, the projected ratios must be capable of being

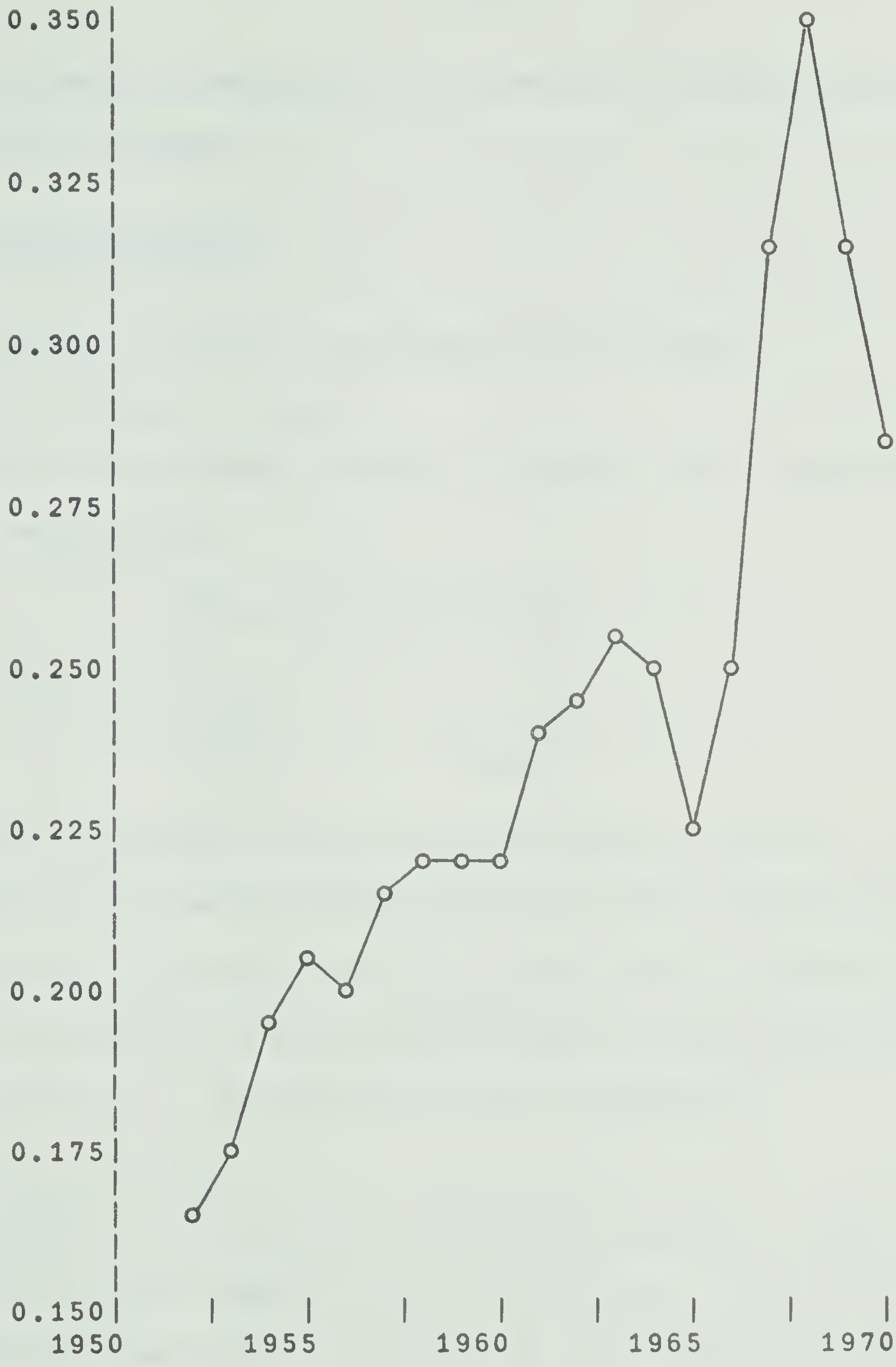

FIGURE 2

Senior Matriculants to Grade XII Ratios

manually modified where external information indicates that such modification is required.

Mathematical Structure

We define a vector of enrolments, E_t, such that

$E_t = (e_1, e_2, \ldots e_{13})_t$

represents the enrolment at time t of grades I to XII and the number of Senior Matriculants.

$$\text{Let } S_t = \begin{bmatrix} s_{1,1} & s_{1,2} & s_{k,1} & s_{13} \\ s_{2,1} & & & \\ s_{13,1} & & & s_{13,13} \end{bmatrix}_t$$

represent the survival ratios from each grade k, at time t, to grade 1, at time t + 1. From the definition of the Cohort Survival Method, all elements of S_t are zero except for 1 equal to k + 1, since the only progression allowed is to the succeeding grade, in the succeeding period.

At time t + 1 the enrolment vector is defined by

$$E_{t+1} = E_t \times S_t \qquad (1)$$

Because of the structure of the matrix S, that is, column 1 containing all zeros, element $(e_1)_{t+1}$ will be zero. This is as it should be since this model does not provide for new entrants into grade I. Further, since we are only interested in projecting Senior Matriculants, the introduction of new entrants is not required unless it is necessary to project beyond twelve years in the future.

Successive iterations will cause further elements to become zero until after 12 iterations, all values are zero, and further projections cannot be made. Provision can be made for longer term projections, by adding a vector of new entrants at each iteration. The model then becomes

$$E_{t+1} = E_t S_t + N_{t+1} \tag{2}$$

where

$$N_{t+1} \equiv (n_1, n_2, \ldots n_{13})$$

represents the vector of new entrants to each grade at each period. Under the strict definition of the Cohort Survival Method, all n are zero with the exception of n_1, ..., that is, entrance is permitted to grade I only.

Determination of the Survival Ratio Matrix

The two conditions for effectiveness which were mentioned earlier, the realistic estimation of future ratios, and the manual modification of these ratios, represent operations on the matrix S_t. Manual modification is easily accomplished by changing the value of the appropriate elements of S_t, at the particular iteration when such changes are expected to occur. For the estimation of future ratios based upon historical data, we have used an exponential-smoothing technique described by Winters.[8]

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[8] Peter R. Winters, "Forecasting Sales by Exponentially Weighted Moving Averages", in Frank M. Bass, et. al., eds., Mathematical Models and Methods in Marketing, (Homewood, Illinois: Richard D. Irwin, Inc., 1961), pp. 492-512.

In this technique, the time series to be analyzed is considered in three parts; a randomly-varying component, a growth or trend component, and a periodically-varying component. The value of each component is estimated from the historical data, using exponentially-weighted moving averages and the components combined to provide the required forecasts. In this application, the periodic component is not used.

If the mean of a random variable is constant, the forecasting problem becomes one of forecasting the expected value of the variable. It has been shown by Winters[9] that this can be estimated by a weighted average of all past observations. The estimate of the mean is given by

$$\tilde{S}_t = aS_t + (1 - a)\,\tilde{S}_{t-1} \tag{3}$$

where

$\tilde{S}_t \equiv$ the estimate of the mean at time t

$S_t \equiv$ the observed value at time t

$a \equiv$ a weighting factor such that $0 \leq a \geq 1$

and the forecast $\hat{S}_{t+1}$ is equal to the mean $\tilde{S}_t$.

The inclusion of a linear growth component requires the estimation of this component and the correction of the estimate of the mean by the addition of the current estimate of the growth. The trend component is an exponentially-weighted average of the difference between successive estimates of the mean, and is given by

$$R_t = B(\tilde{S}_t - \tilde{S}_{t-1}) + (1 - B)R_{t-1} \tag{4}$$

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[9]Ibid., pp. 494-500.

where

$R_t \equiv$ the estimate of the trend at time t

B $\equiv$ a weighting factor such that $0 \leq B \geq 1$

Equation (3) then becomes:

$$\tilde{S}_t = aS_t + (1 - a)\,(S_{t-1} + R_{t-1}), \qquad (5)$$

and the forecast of the future value of S, T periods in the future is

$$\hat{S}_{T,t} = \tilde{S}_t + T \times R_t \qquad (6)$$

The estimate of the future enrolment can be derived, therefore, from the equation obtained by substitution of equation (6) for S in equation (1)

$$\begin{aligned}\hat{E}_{t+1} &= E_t\hat{S}_t \\ &= E_t\,(\tilde{S}_{t-1} + R_{t-1}) \qquad (T = 1)\end{aligned} \qquad (7)$$

Note: although the enrolment at time t is known, the survival ratio from t to t+1 is the forecast made at t - 1, for one period in the future.

The enrolment forecast for the following period is:

$$\begin{aligned}E_{t+2} &= E_{t+1}\,(S_{t-1} + 2R_{t-1}), \quad (T = 2) \\ &= E_t\,(\tilde{S}_{t-1} + R_{t-1})\,(\tilde{S}_{t-1} + 2R_{t-1})\end{aligned} \qquad (8)$$

and the general formula for any year is

$$E_{t+T} = E_t \prod_{i=1}^{T} (\tilde{S}_{t-1} + iR_{t-1}), \quad (T \leq 12) \qquad (9)$$

where

$$\tilde{S}_{t-1} = aS_{t-1} + (1 - a)\,(\tilde{S}_{t-2} + R_{t-1}) \qquad (5a)$$

$$R_{t-1} = B(\tilde{S}_{t-1} - \tilde{S}_{t-1}) + (1 - B)\,R_{t-2} \qquad (4a)$$

The major advantage of using this method is derived from two factors. Firstly, the only data which must be carried from period to period

are the latest estimate of trend and mean. Secondly, the estimates are continuously updated as new information becomes available and the model will correct for changes in the actual values of the mean and trend at a rate dependent upon the value of the weighting constants used. Further, when future changes are known in advance, the stored estimate can be modified to reflect the effect of the changes, and provide faster stabilization to the new level.

Selection of Smoothing Constants

Implementation of this forecasting model requires the determination of appropriate values of the weighting constants a and B. The method given by Winters[10] determines those weights that minimize some measure of the forecast error and uses these in the smoothing model. The appropriate measure is determined by the behaviour of the costs associated with over- and under-estimating the actual enrolment.

Cost behaviour is established by the institutional environment, i.e., the method of funding, the degree of unused capacity, and the nature of the student demand for places; and whether the forecasts are used for short-term operational planning, or longer-term capital planning.

In the short run, the costs of overestimating consist mainly of excess faculty salaries, which may be offset by benefits derived in other areas, increased faculty research for example. It can also be argued that a social benefit is derived from am improved instructional milieu resulting from the lowered student-faculty ratio. Other

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[10] Ibid., pp. 500-510.

potential short-term costs of overestimating are public and political feelings of academic inefficiency, and subsequent reluctance to provide funds in future periods. Underestimating in the short term incurs the cost of high student-faculty ratios with their attendant faculty and student dissatisfaction, and the possibility of the rejection of potential students. From an economic point of view, the objective, therefore, should be to minimize the average overestimation error, whereas, the underestimation error should be minimized from an academic viewpoint.

In the long run, similar effects occur, but the time required to correct the effects of estimation error is significantly increased, and the cumulative cost of an estimation error is greater in both economic and academic terms.

For a number of reasons, including the difficulty in assessing the comparative value of non-economic costs, it has been decided that this implementation will be based upon the assumption that the costs of overestimating equal those of underestimating, and further, that the loss function is quadratic in nature. That is, a large error is worse than an equal sum of small errors. The statistic which was chosen for minimization was the standard error of forecast defined as:

$$SFE = \sqrt{\frac{\sum_{t=1}^{N} (\hat{S}_t - S_t)^2}{N - 1}} \qquad (10)$$

where

$\hat{S}_t \equiv$ the forecast for period t made at t - 1

$S_t \equiv$ the observed value at period t

N $\equiv$ the number of observations

This statistic is appropriate for the short-run use of the model, given the assumptions which have been stated. For long-run purposes, however, a more appropriate statistic would be based upon the maximum error between the actual observations and the sets of forecasts made at each period, for the planning horizon. For example, if a five-year planning horizon is used, then at each year five forecasts are made, one for each year in the planning period. The maximum error in this set, in conjunction with the maximum errors of the forecasts made at other periods is used to generate the long-term error statistic. This statistic is described by:

$$SFE = \sqrt{\frac{\sum_{t=1}^{(N-T)} (\underset{i}{\mathrm{Max}} \{\hat{S}_{t,i} - S_{t+i}\})^2}{N - T - i}} \qquad i = 1, 2, 3, \ldots T \quad (11)$$

where

$\hat{S}_{t,i} \equiv$ the forecast made at t, i periods into the future

$S_{t+1} \equiv$ the actual observation for time t + i

T $\equiv$ the planning horizon

N $\equiv$ the number of observations available

The limited number of observations (nineteen) which were available for the high school model precluded the use of this method, as a smooth contour of forecast errors could not be developed, and a subjective selection of smoothing constants was made. Brown[11] has shown that there

[11]Robert Goodell Brown, Smoothing, Forecasting, and Prediction of Discrete Time Series, (Englewood Cliffs, N. J. : Prentice Hall, Inc. 1962), p. 108.

is a relationship between the value of the smoothing constant and an equivalent number of observations in a moving average, as shown in Table II.

Table II

Smoothing Constants and Equivalent Moving Averages

Number of Observations in a Moving Average	Smoothing Constant
3	0.500
4	0.400
5	0.333
5.67	0.300
6	0.286
9	0.200
12	0.154
18	0.105
19	0.100
24	0.080
39	0.505
52	0.038
199	0.010

Since we only have 19 observations, and we wish to provide at least the equivalent of three years damping of variations, the range of usable weighting factors is from 0.1 to 0.5.

It is expected also that survival ratios as such should level off to a relatively constant value as time progresses, (assuming a stable pattern of migration) and that more recent observations are more

valuable than the older ones. For these reasons, weights of 0.3 were chosen for all grades for both the mean and trend estimates.

For the Senior Matriculant ratios, weighting factors of 0.5 for the mean estimate and 0.2 for the trend estimate were used, on the basis that the discontinuity which occurred in the period from 1966 to 1967 required a heavier weighting of current observations in the estimate of the mean, but a lighter weighting of current observations in the long-term growth trend, which is expected to continue once the perturbations of the discontinuity have subsided.

The University Enrolment Model

A transition coefficient model has been developed for the analysis and projection of student flow through the university. However, an intermediate step must be taken between the projection of high school matriculants and the projection of students' flow through the university. That is, the matriculants must be allocated as new entrants to each of the state classifications in the model. In addition, new entrants from other sources must be projected and allocated. Discussion of the basis and method of these allocations will be left until later.

The classification structure of the model has been determined by the availability of historical data. Although this does not represent the ideal structure for administrative planning, it does provide for a usable model during the interim period in which the necessary data for an alternative structure is being collected. Since the classification structure does not change the fundamental nature of the model, only its size and parameter values, the initial model development is not wasted.

Machine readable data for students' flow is available for the past four years. However, because of format and informational content changes from year to year, some data are not easily recovered. For this reason, a simple classification structure consisting of the faculty and program year of registration was used for those states within the university itself. In addition, a number of external states were included to provide for the termination of formal education at several levels of achievement and to allow for intermediate periods in which a student might be temporarily out of the system.

The Mathematical Structure[12]

Let $E_t \equiv (e_1, e_2, \dots e_n)_t$

represent the vector of student populations in all states at time t and let

$N_t \equiv (n_1, n_2, \dots n_n)_t$

represent the vector of new students entering all states at time t

$$C_t \equiv \begin{bmatrix} c_{11}, c_{12}, c_{kl}, \cdots c_{1n} \\ cn_1, \dots\dots\dots\dots\dots c_{nn} \end{bmatrix}_t$$

defines the matrix of coefficients of transition from any state k at time t to any state l at time t + 1.

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[12]The mathematical structure of this model is similar to that of Baisuch and Wallace, "A Computer Simulation Approach to Enrollment Projection in Higher Education".

Then

$$E_{t+1} = E_t C_t + N_{t+1} \tag{12}$$

represents the vector of populations one period later.

The population vector two periods later is

$$\begin{aligned} E_{t+2} &= E_{t+1} C_{t+1} + N_{t+2} \\ &= (E_t C_t + N_{t+1}) C_{t+1} + N_{t+2} \end{aligned} \tag{13}$$

The general formula for the population T periods in the future is

$$\begin{aligned} E_{t+T} &= E_{t+T-1} C_{t+T-1} + N_{t+T} \\ &= (E_{t+T-2} C_{t+T-2} + N_{t+T-1})\, C_{t+T-1} + N_{t+T} \\ &= \qquad \vdots \\ &= (((E_t C_t + N_{t+1})\, C_{t+1} + N_{t+2})\, C_{t+2} + \ldots + N_{t+T-1}) \\ &\qquad C_{t+T-1} + N_{t+T} \\ &= E_t \times \prod_{i=0}^{T-1} C_{t+i} + \sum_{i=1}^{T-1} (N_{t+i} \times \prod_{k-i}^{T-1} C_{t+k}) + N_{t+T} \end{aligned} \tag{14}$$

Given the vector of enrolments (E_t) at time t, the projected enrolments are determined by the intervening values of the transition coefficients and the new entrants vectors. Since the number of coefficients is directly proportional to the square of the number of classification states in the system, the matrix becomes very large for relatively simple classification structures. In practice, however, the task of projections is not as difficult as it first appears. The matrix may be considered as

$$C \equiv \begin{array}{c|c} I & 0 \\ \hline A & B \end{array}$$

where

$I \equiv$ (m x m) identity matrix, where m is the number of absorbing states

$0 \equiv$ (m x (n-m)) zero matrix

$A \equiv$ ((n-m) x m) matrix of coefficients of transition from active states to absorbing states

$B \equiv$ ((n-m) x (n-m)) matrix of transition coefficients between active states.

Since transition to any given state is not possible or feasible from certain other states, many coefficients can be immediately identified as zero.

Projection of the Transition Coefficients

Because of the limited historical data available at this time, an exhaustive search for the most appropriate projection method would make little sense. Since exponential smoothing programs were developed for the high school graduate model, they were included in the university model.

Recalling equation (6)

$$\hat{S}_{t+T} = \tilde{S}_t + TR_t \qquad (6)$$

Where

$\hat{S}_{t+T} \equiv$ the forecast value of S, T period in the future, made at time t

$\tilde{S}_t \equiv$ the estimate of the mean at time t

$R_t \equiv$ the estimate of the trend at time t

Changing notation slightly and substituting (6) in (14) for the estimate of C

$$E_{t+T} = E_t \prod_{i=1}^{T-1} (C_{t-1} + iR_{t-1}) + \sum_{i=1}^{T-1} (N_{t+1} \prod_{p=i}^{T-1} (C_{t-1} + pR_{t-1})) + N_{t+T} \qquad (15)$$

where C_{t-1} is the estimate of the mean of the transition coefficient made at t -1,

and R_{t-1} is the estimate made at t - 1 of the trend with time of the coefficient.

Since the exponential smoothing method always requires a preceding estimate of both mean and trend, initial values must be used to start the process. Where historical data are available, these values may be derived by starting with a subjective or analytical estimate of the values and then smoothing the historical data to obtain the current estimates. The degree to which the effect of errors in the initial estimates is washed out will be a function of the smoothing constants chosen for the process. With low values of smoothing constants, the initial estimates will have long-term effects, but will be quickly eliminated with high values.

If a simple regression analysis with respect to time is used to establish the initial values of mean and trend, and smoothing constants are set at zero, the projection of coefficients will be on the original regression line regardless of the degree to which subsequent actual data depart from this line. Where there are sufficient historical data to develop an accurate regression line and where the process is expected to behave the same in the future as in the past, the exponential

smoothing characteristics of the model are not required, and the model can use the initial estimates until such time as these are changed. In the case where there is only a short history, the ability of the model to modify the initial estimates as more data become available, provides an administrative tool which does not require a major technical reevaluation of its parameters each time that new data are received.

A logical extension of this model is the development of a technique by which the smoothing constants may be modified as time progresses. Some general work has been done in this area, but it is felt that the application of these methods is beyond the scope of the present project.

Determination of the New Entrants Allocations

New entrants to the university system are classified according to their source, either from the Alberta Senior Matriculant body or from all other sources. The input from sources other than the Senior Matriculants has been comparatively stable and of small size. The mean of the historical inputs to each state from this source is used as the forecast input.

Simple linear regression has been used, as an interim measure, to determine the relationship between the Senior Matriculant body and the possible entrance states in the university. As soon as the university model is complete, however, the Senior Matriculants will be incorporated as a number of states in the model with their associated

transition coefficients. This will allow for the provision of a varying time delay between high school graduation and university entrance, to accommodate those students who do not enter directly after high school.

CHAPTER III

Evaluation of the Model

The evaluation of any model in the long run is based upon the validity of the information provided by the model when compared to the actual case. In a projection model, such testing can be accomplished by reserving some historical data from the initial parameter determination and using these data as benchmarks for evaluating the model's projection accuracy. Where a paucity of historical information exists, validity testing must be deferred until future information becomes available. In either case, a wide range of statistical tests are available to measure the significance of projection errors.

Initial tests can be made even before the model is implemented. These can be broadly classified as content evaluation tests and structural evaluation tests.

Content Evaluation

The fundamental problem of content testing is the subjective evaluation of the degree to which the model contains, or provides for, all underlying factors in the real world which affect the process being modeled. For the university transition coefficient model, the number of states can be made large enough to accomodate any and all combinations of variations in process characteristics, at least in theory.

The classification system which is chosen in practice will limit the degree to which the model is representative, and it is the classification system itself which must be evaluated. From a very pragmatic point of view, the system which has been chosen is best suited to the needs of the university, given the constraints which are imposed by the availability of usable data, and by the resources which are available at this time. This does not imply that there are no better classification systems, and it is hoped that, as data become available, improvements will be made.

The preceding comments apply to the high-school graduate model in even greater degree, as the classification structure imposed by the cohort survival technique is more restrictive. Given that more data were available, however, the model could accomodate alternative classification structures, since it has been designed as a special case of the more general transition coefficient model.

Structural Evaluation

Structural evaluation is the measurement of reality of the relationships between variables which are included in the model. Two main areas of concern are present: the nature of the transition coefficients themselves, and the projection of the coefficients and new entrants.

The use of transition coefficients implies a regularity of student movement which may not exist. In particular, the use of a single valued transition probability between any two states at any time,

implies a deterministic attribute which is unlikely to be realistic. Intuitively, however, regularity of student progress does exist in most cases, and the number of occurrences of totally random coefficients is expected to be low. Further, if the method by which the coefficients are projected provides an unbiased estimate of the mean of the distribution of coefficient, this second criticism is removed for single-valued population projections.

Testing of the reality of the projections of the coefficients and new entrants, as a function of time, is hampered by the same restrictions as the validity testing of the overall model. That is, the goodness of fit determination is limited by the availability of benchmark data. If regression techniques have been used to project these variables, some measure of historical accuracy is available. The degree to which this accuracy is representative of the forecast accuracy will be determined by the stability of the process being modelled.

The overall evaluation of this implementation of the model will be based upon the degree of utility provided by the model in the university planning process, and as such must be deferred until sufficient experience has been gained in its use. Some preliminary forecasts have been made of the total non-professional undergraduate body; however, full implementation of the model has not been achieved as yet.

CHAPTER IV

Results

The high-school graduate model has been implemented in an interactive remote terminal mode on the IBM 360/67 computer, using APL as the programming language. This language is particularly suited to this type of model because of the ease with which it handles matrix and vector operations.

The implementation provides for the revision of the estimates of the mean and trend of each survival ratio, at any time, and allows the inclusion of new data as it becomes available. The model projects the school enrolments and the numbers of Senior Matriculants for each year over the period specified by the operator. As an interim measure, the first year non-professional undergraduate university enrolment is also projected using the relationship established by simple linear regression. A typical session is shown in Figure 3. In this session, the estimate of the mean for the Senior Matriculant to grade XII ratio is revised to accomodate the effects of known policy change between 1966 and 1967, and the subsequent Matriculant data are entered. Seven years of projected Senior Matriculants and first-year enrolments are given.

Appendix B contains graphs of the actual historical data, and the corresponding forecasts made, using the exponential smoothing technique, in the previous period.

PROJECT

IN RESPONSE TO QUERIES, ANSWER YES OR NO.
DO YOU WISH TO REVISE THE ESTIMATES OF MEAN OR TREND?
YES
ENTER THE NEW ESTIMATES GIVING THE RATIO NUMBER
(I.E. 1 FOR THE RATIO FROM GRADE 1 TO GRADE 2, ETC.),
THE NEW ESTIMATE FOR THE MEAN AND THE NEW ESTIMATE FOR
THE TREND. USE RATIO NUMBER 12 FOR GRADE 12 TO SR.
MATRICULANTS RATIO. ENTER THE DATA FOR EACH RATIO
ON ONE LINE, SEPARATED BY BLANKS OR COMMAS. WHEN
THERE ARE NO MORE DATA, ENTER A ZERO.
⎕:
12 0.313 0.0029
⎕:
0
ANY NEW ENROLMENT DATA (EXCLUDING MATRICS.)?
NO
ENTER THE NUMBER OF PERIODS FOR WHICH ENROL-
MENT PROJECTIONS ARE REQUIRED (11 OR LESS).
⎕:
7
THE PROJECTED SCHOOL ENROLMENTS ARE:

YEAR	1	2	3	4	5	6	7	8	9	10	11	12
1971	0	38660	36924	37083	36394	36371	36265	34026	32594	32262	29213	29283
1972	0	0	37715	36211	36443	36205	37184	35443	32826	33142	30530	31909
1973	0	0	0	36624	35544	36292	36914	36378	33883	34337	31328	33125
1974	0	0	0	0	35908	35434	36901	36151	34459	36432	32423	33762
1975	0	0	0	0	0	35835	35930	36175	33929	38057	34363	34706
1976	0	0	0	0	0	0	36236	35259	33635	38463	35857	36532
1977	0	0	0	0	0	0	0	35595	32475	39112	36200	37859

ZERO ENROLMENT IS SHOWN WHERE PROJECTIONS CANNOT BE MADE
USING THIS METHOD.

ANY NEW DATA ON SENIOR MATRICULANTS?
YES
ENTER NEW MATRICULANT DATA GIVING THE YEAR
AND THE NUMBER OF MATRICULANTS. WHEN THERE ARE NO MORE
DATA, ENTER A ZERO.
⎕:
1967 6886
⎕:
1968 7819
⎕:
1969 7910
⎕:
1970 7787
⎕:
0
THE PROJECTED MATRICULANT NUMBERS ARE:

YEAR	NUMBER
1971	8873
1972	9489
1973	9665
1974	9661
1975	9737
1976	10044
1977	10196

THE PROJECTED FIRST YEAR UNIVERSITY ENROLMENTS ARE:

YEAR	NUMBER
1972	4941
1973	5249
1974	5337
1975	5335
1976	5373
1977	5527
1978	5603

FIGURE 3

Sample Terminal Session

These plots give a general impression of the degree to which the forecast changes as a result of changes in the observed value and provide a basis for a subjective evaluation of the tracking ability of the model. It should be noted, however, that the apparent immediate response to the step increase in the Senior Matriculant ratio shown in Figure 28 is a result of the manual adjustment previously mentioned. In all other cases, no adjustments have been made.

Comparing the projections of high school enrolments shown in Figure 3 with those made by Hansen[13] for 1971-72 and 1975-76, shows three discrepancies.

The model projection for grade X is the same in Hansen's for 1971-72 at a level of 33,000 students. By 1975-76, however, the model projects 39,000 students compared to 36,000. At the same time, it shows the grade IX level staying constant at 33,000 compared to Hansen's growth from 35,000 to 37,000. The actual survival ratio from grade IX to grade X has shown a steady increase over the period for which we have data, with a more rapid increase in the immediate past. This growth trend in the ratio is continued in the projection, to the extent that the projected grade X enrolment is significantly greater than that in grade IX.

The third discrepancy is a constant difference of about 3,000 students in the grade XII population, with the model projection being greater. This can be accounted for by the fact that the Department

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[13]Hansen, op. cit., Table 19, p.38.

of Education statistics, upon which the model is based, include such categories as adult evening credit and part-time enrolments.

The combined effect of these discrepancies is relatively small, giving total grades IX to XII enrolments that differ by less than two percent.

The general university transition coefficient model has not been implemented to date for two reasons. Although the classification structure has been limited to a simple faculty-program year of registration scheme; with a number of input, output, and waiting states; the resultant coefficient matrix contains approximately 10,000 elements. The present APL workspaces cannot handle a matrix of this size, therefore this model must be written in another language. This will also provide a more flexible means of data input and output, and greater transportability of the model, as the number of APL installations is relatively small. Secondly, the collection of historical data for coefficient determination is not complete.

Since we wish to retain the interactive capabilities of the existing model, the new language must be one which is supported by the Michigan Time Sharing System presently in use at the university.

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APPENDIX A

TABLE III

Elementary and Secondary Grade Enrolments*

Year	Grade											
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1952	22,416	20,096	20,180	18,678	17,601	16,684	15,567	13,958	12,288	9,009	7,056	6,158
1953	25,353	20,494	20,383	20,164	18,551	17,505	16,448	14,482	12,648	9,637	7,199	6,217
1954	27,939	23,464	20,678	20,326	20,037	18,555	17,408	15,461	13,386	9,919	7,902	6,345
1955	26,413	26,649	23,310	20,469	20,219	19,974	18,482	16,441	14,549	10,843	8,472	6,884
1956	27,188	25,906	26,428	23,065	20,372	20,103	19,810	17,412	15,593	11,724	9,028	7,320
1957	27,397	26,222	26,050	26,212	22,958	20,272	20,021	18,735	16,472	12,670	9,665	7,723
1958	28,955	26,679	26,291	25,859	26,026	22,799	20,309	19,213	18,114	13,738	10,780	8,456
1959	30,716	28,112	26,895	26,061	26,056	25,631	22,944	19,569	18,586	15,227	12,033	9,724
1960	32,536	30,105	28,378	27,042	26,211	25,903	26,020	22,222	19,161	15,707	13,344	11,291
1961	34,520	31,765	30,346	28,166	27,070	26,019	26,409	25,042	21,757	16,097	14,021	13,223
1962	35,555	33,765	31,160	29,864	27,960	26,355	26,371	25,310	24,492	18,204	14,506	14,160
1963	35,257	34,908	33,048	31,407	29,980	27,775	27,123	25,537	25,104	20,799	16,597	14,692
1964	36,554	34,504	34,221	32,918	31,319	29,612	28,485	26,219	25,319	21,490	19,314	16,697
1965	37,241	35,964	33,553	34,188	32,745	30,931	30,185	27,576	26,034	22,116	20,201	20,172
1966	38,160	36,507	34,708	33,650	33,779	32,270	31,497	29,118	27,618	22,696	20,374	21,781
1967	38,441	37,395	35,694	35,046	33,277	33,205	33,056	30,019	29,181	24,646	20,963	21,970
1968	38,550	38,277	36,715	37,084	34,408	33,404	34,239	32,336	29,491	27,172	22,813	22,484
1969	39,178	38,344	38,064	37,041	35,666	34,240	34,397	33,767	31,163	28,559	25,941	25,227
1970	39,567	37,850	37,445	36,991	36,599	35,375	34,851	33,582	32,667	30,837	26,631	27,138

*Source: Government of Alberta, Department of Education

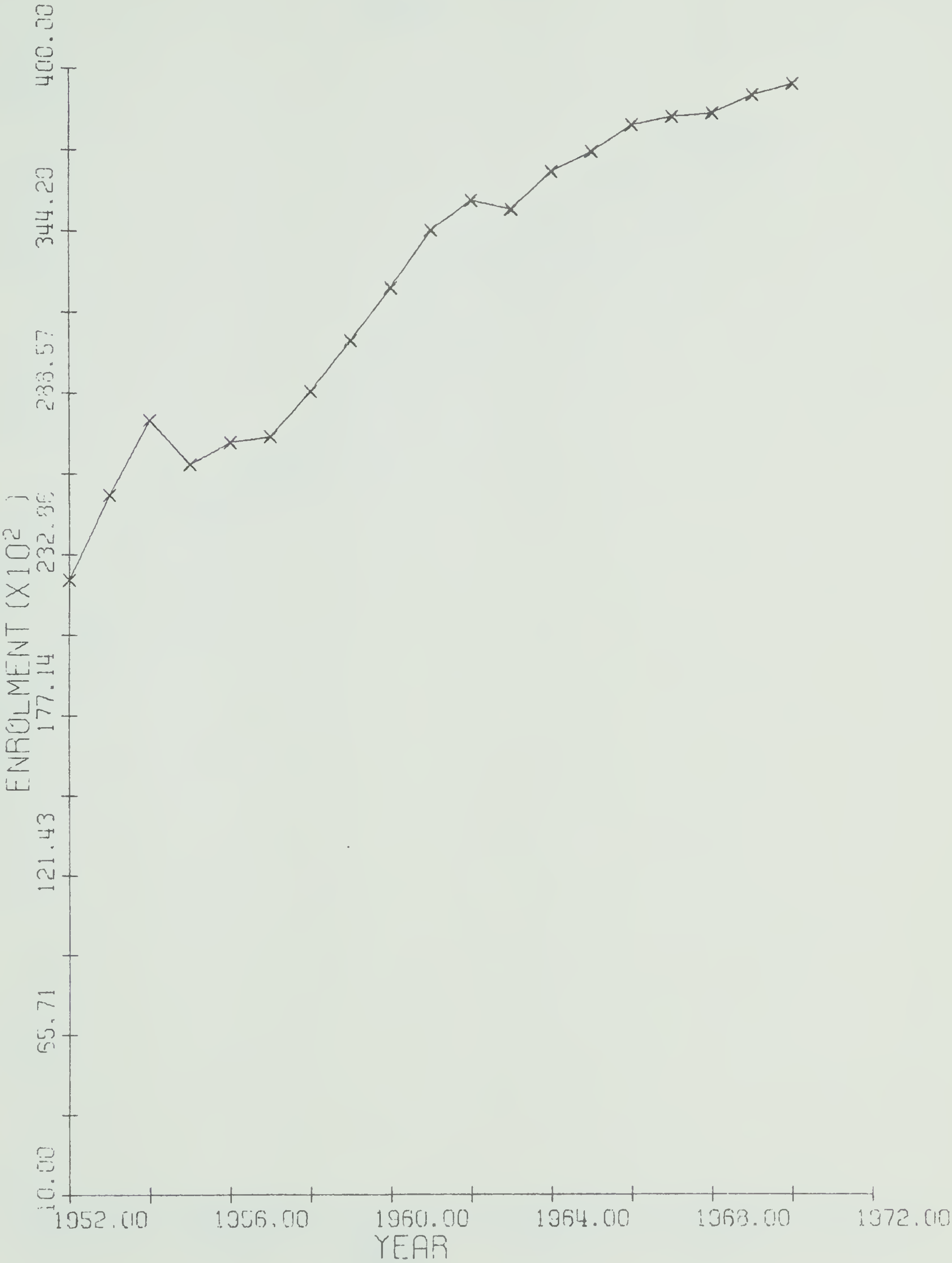

FIGURE 4

Grade I Enrolment by Year

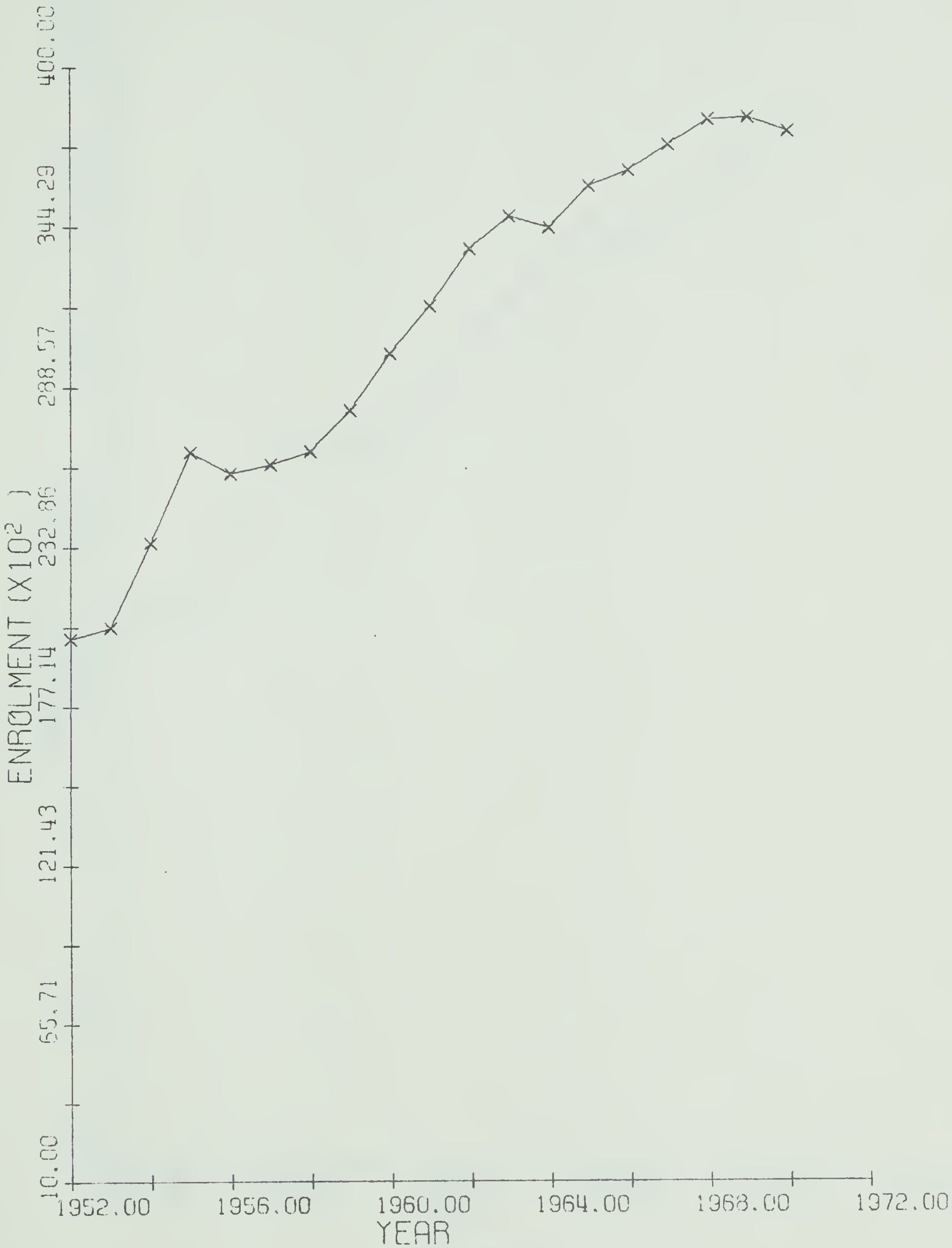

FIGURE 5

Grade II Enrolment by Year

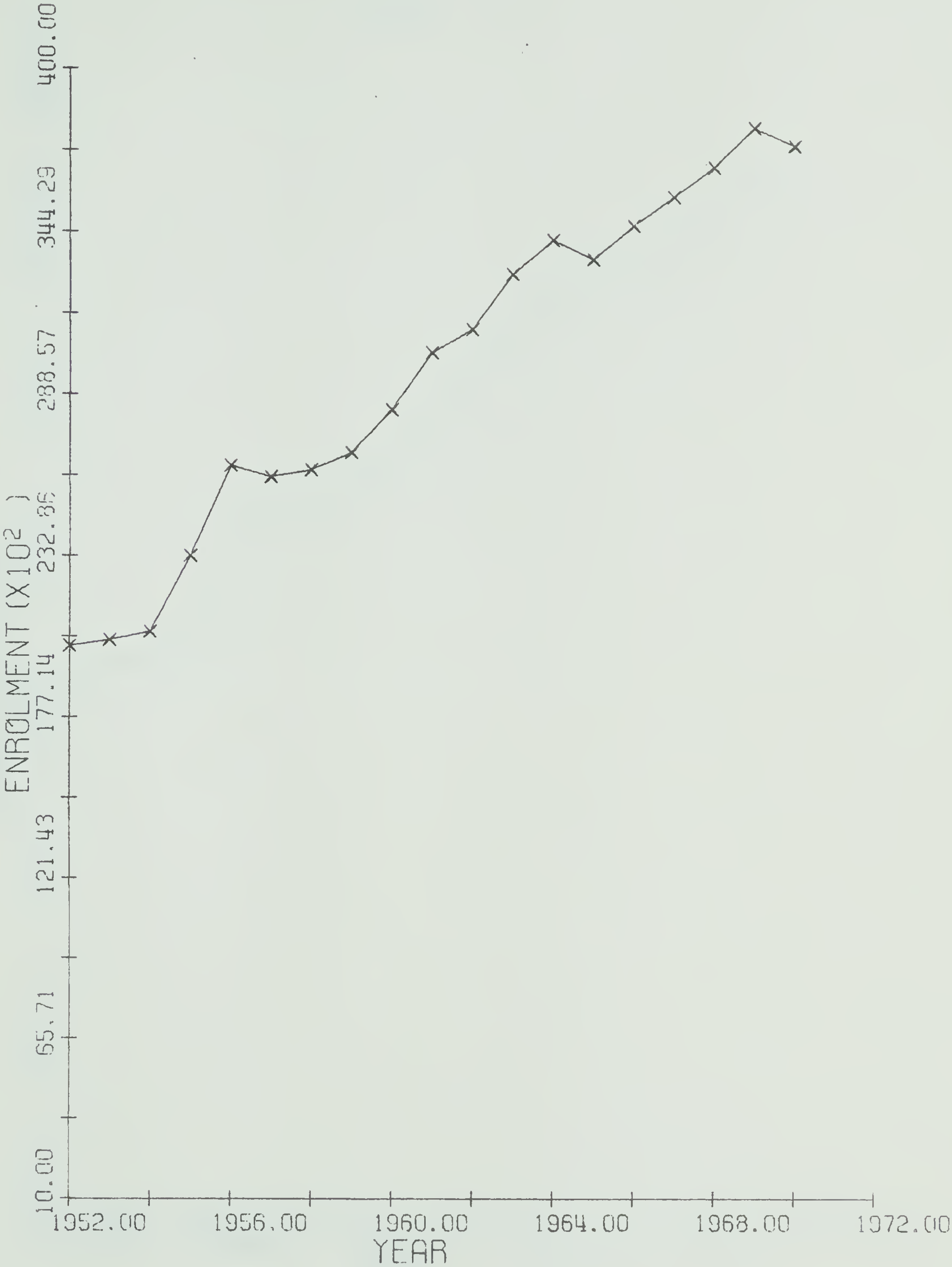

FIGURE 6

Grade III Enrolment by Year

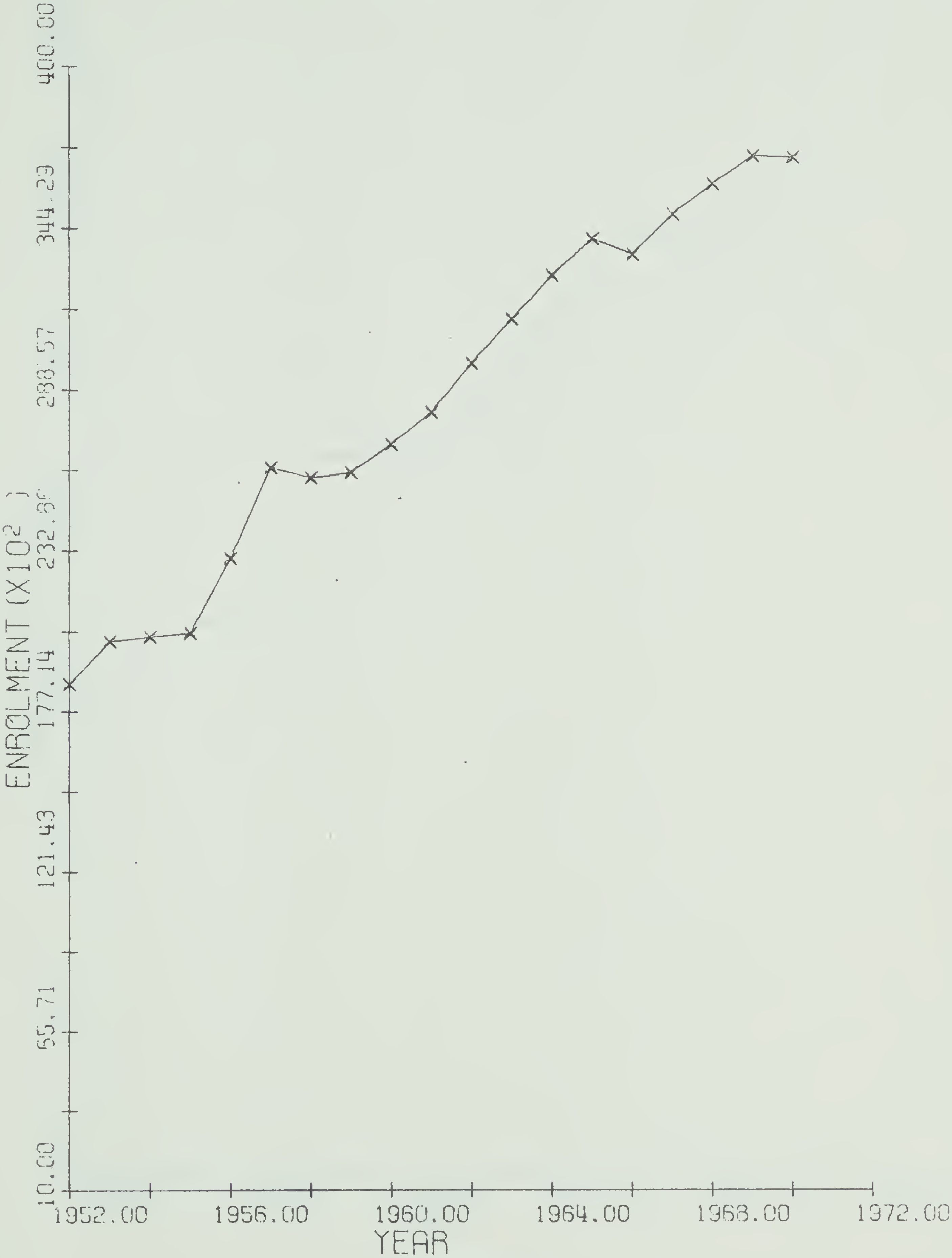

FIGURE 7

Grade IV Enrolment by Year

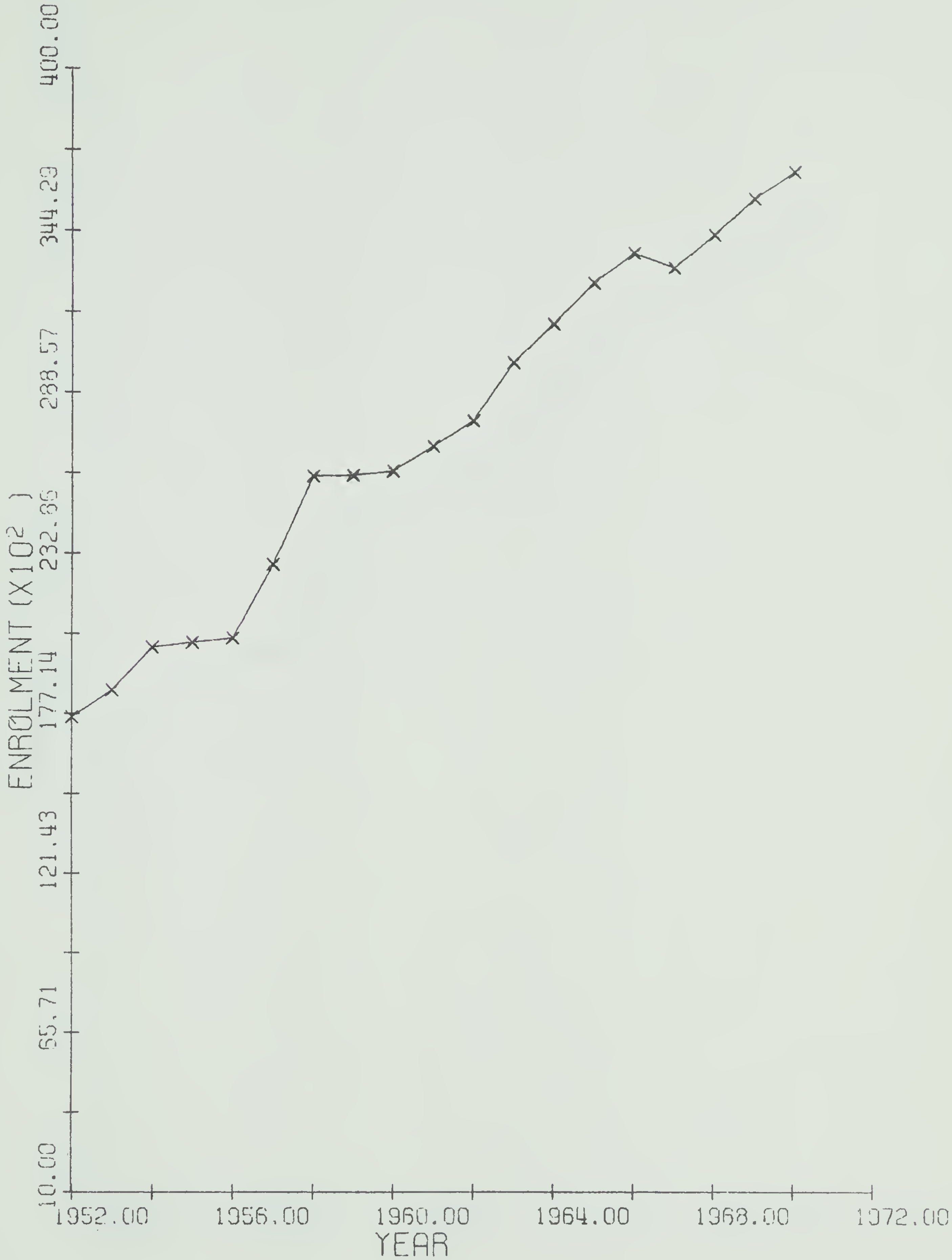

FIGURE 8

Grade V Enrolment by Year

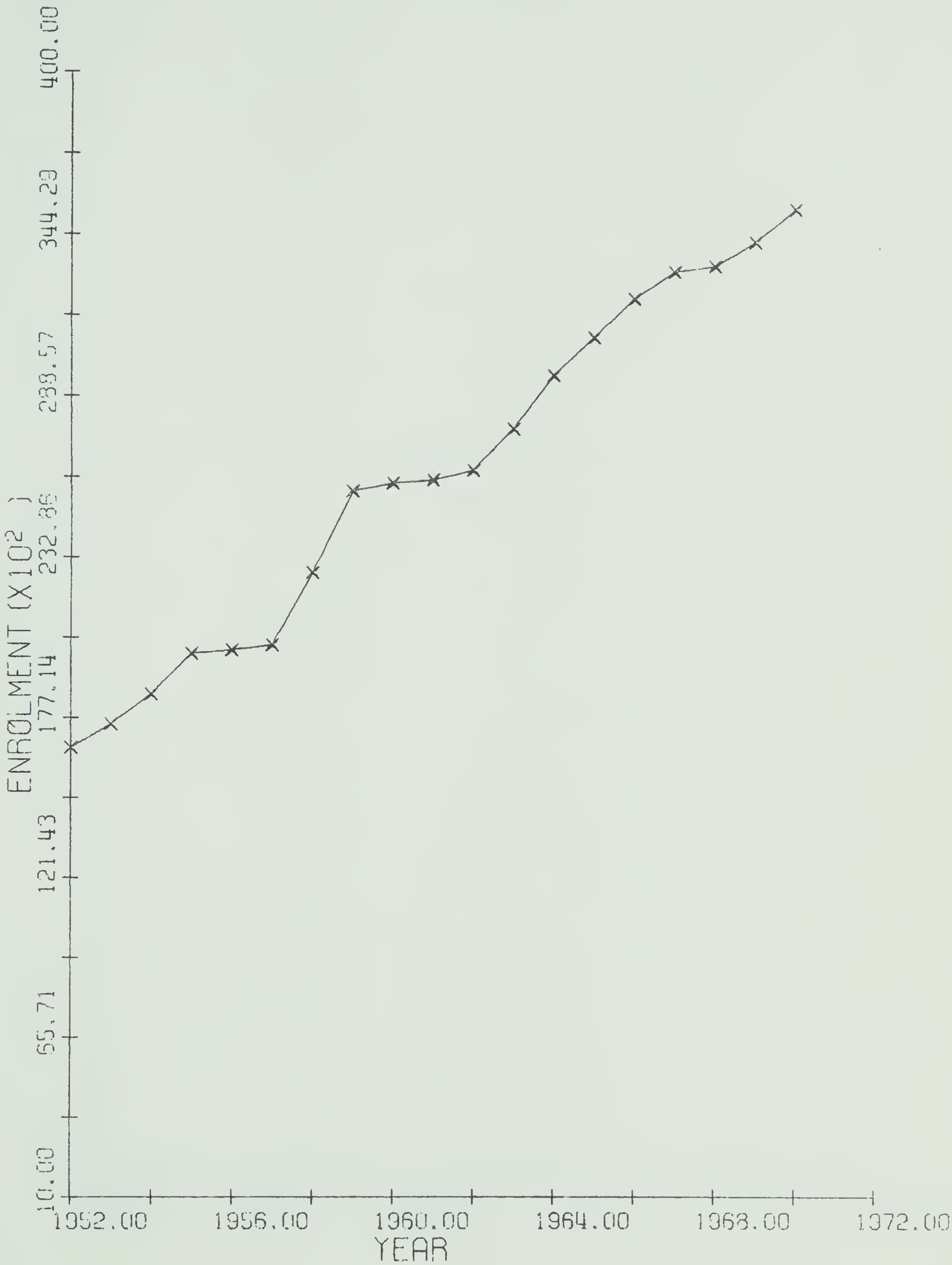

FIGURE 9

Grade VI Enrolment by Year

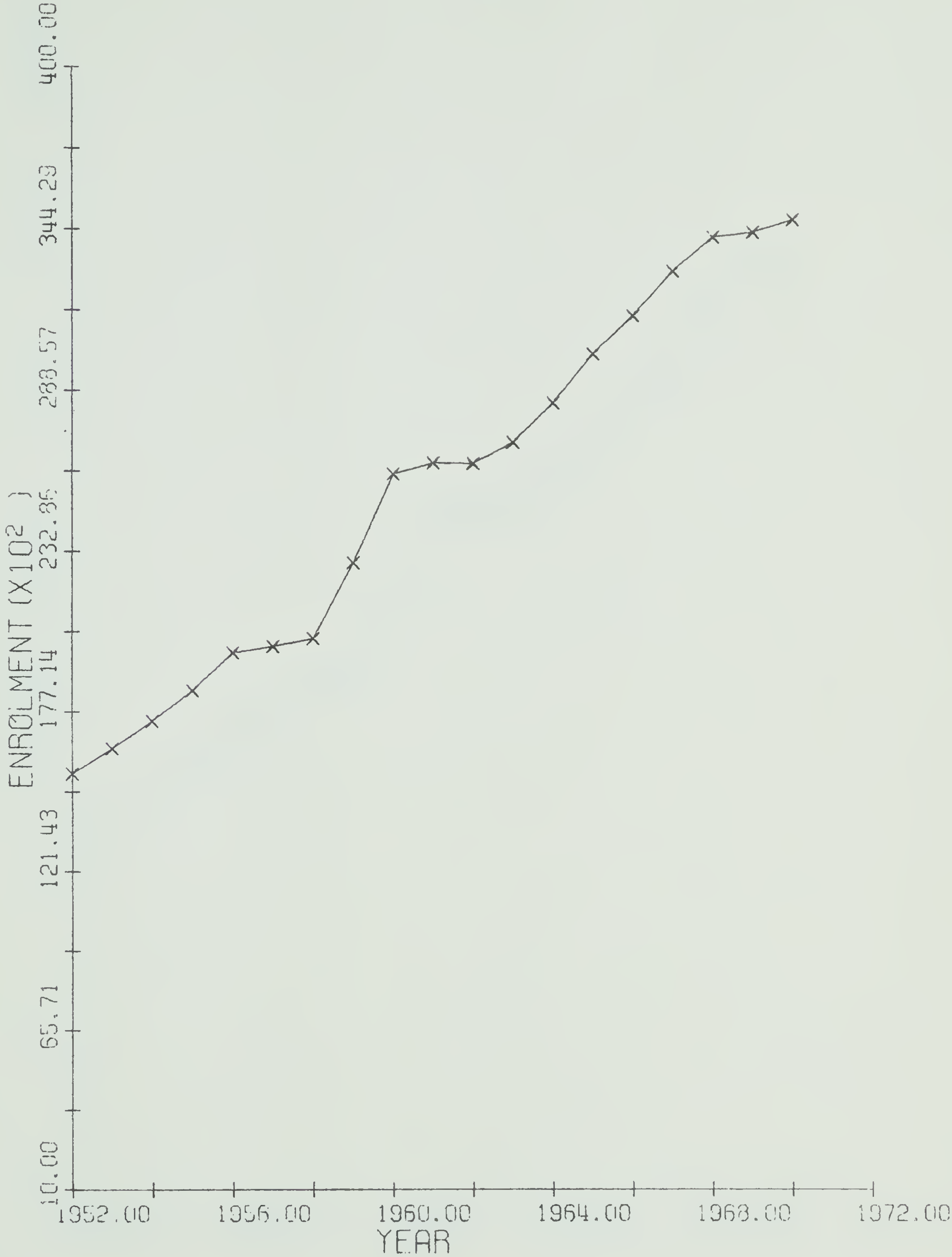

FIGURE 10

Grade VII Enrolment by Year

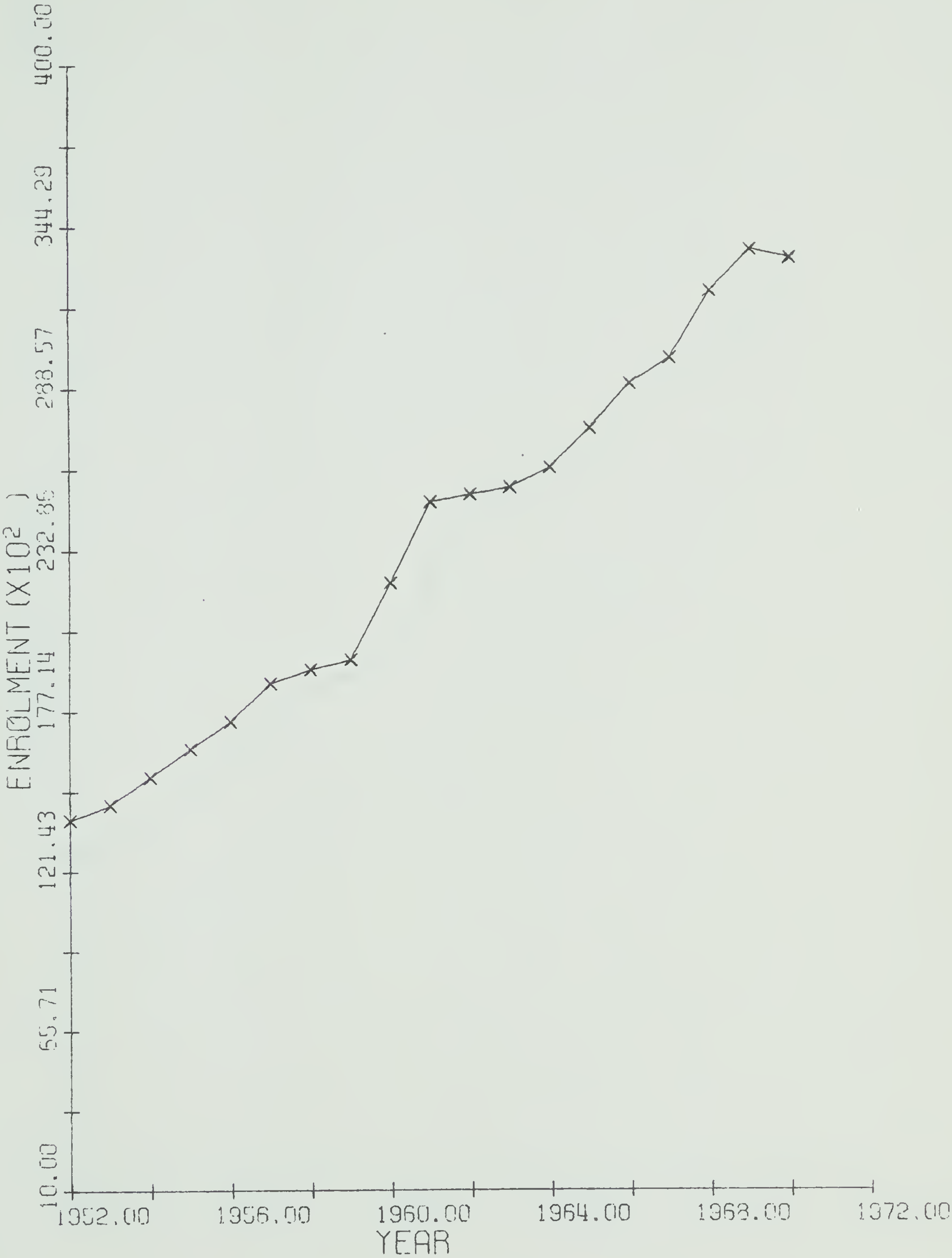

FIGURE 11

Grade VIII Enrolment by Year

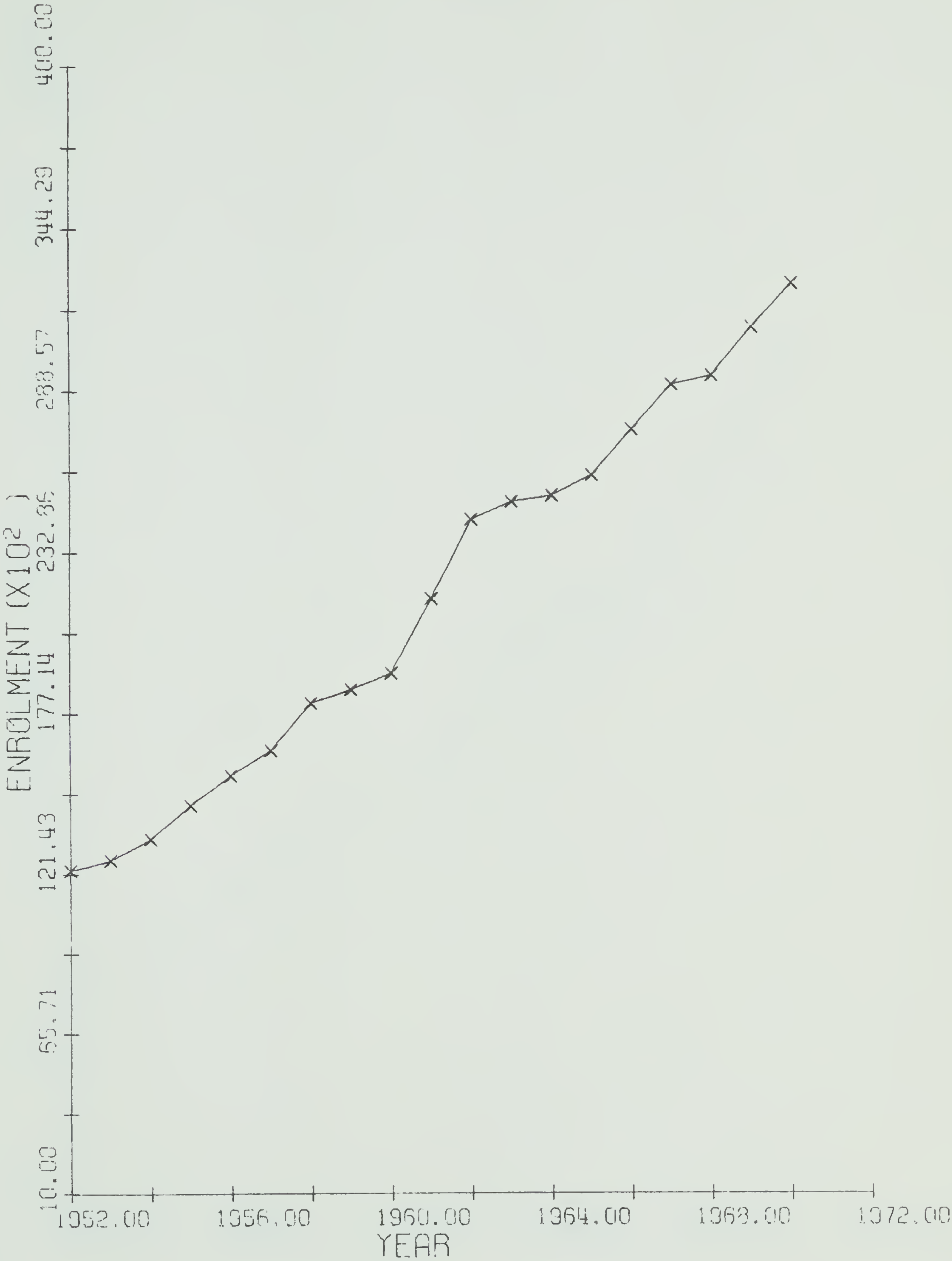

FIGURE 12

Grade IX Enrolment by Year

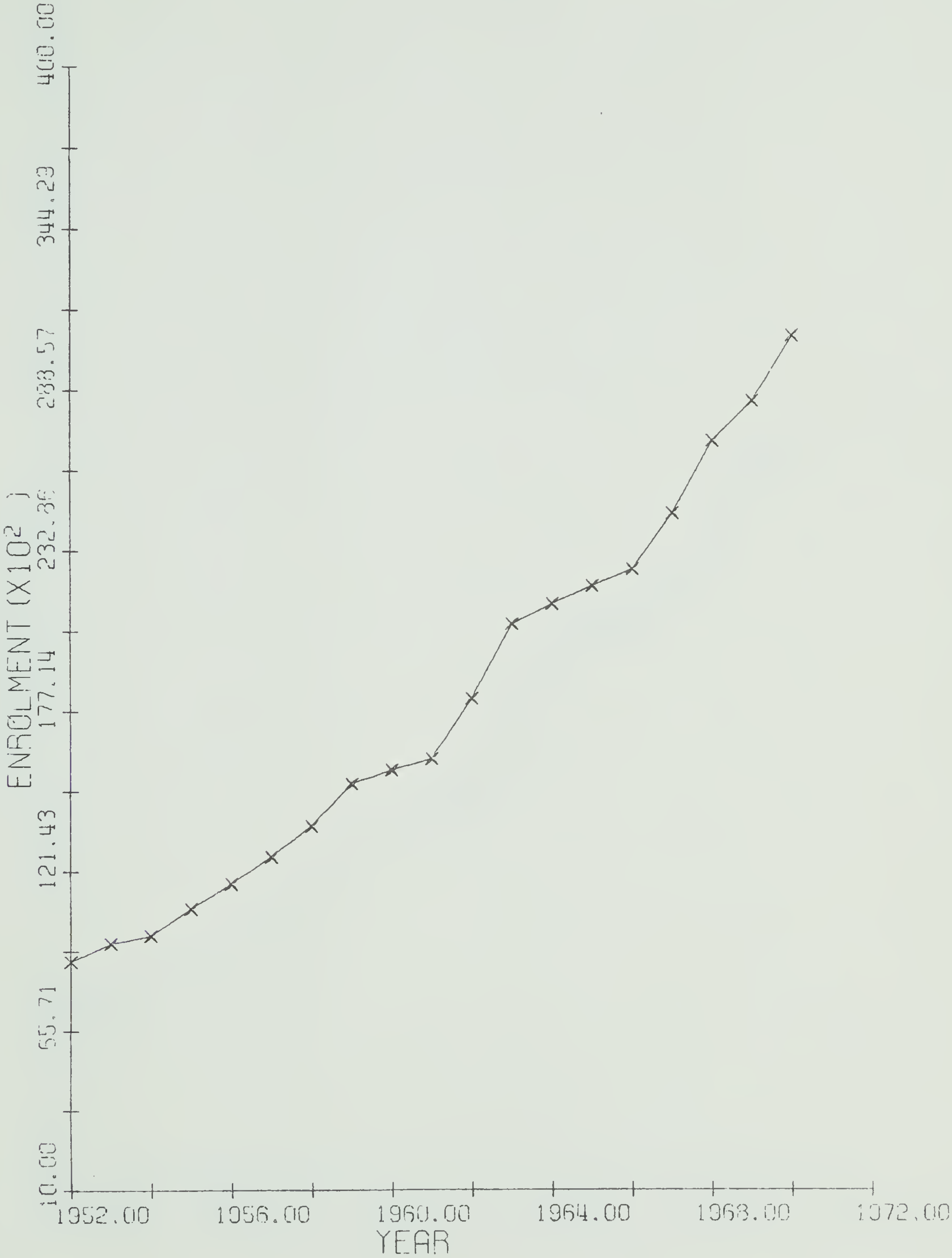

FIGURE 13

Grade X Enrolment by Year

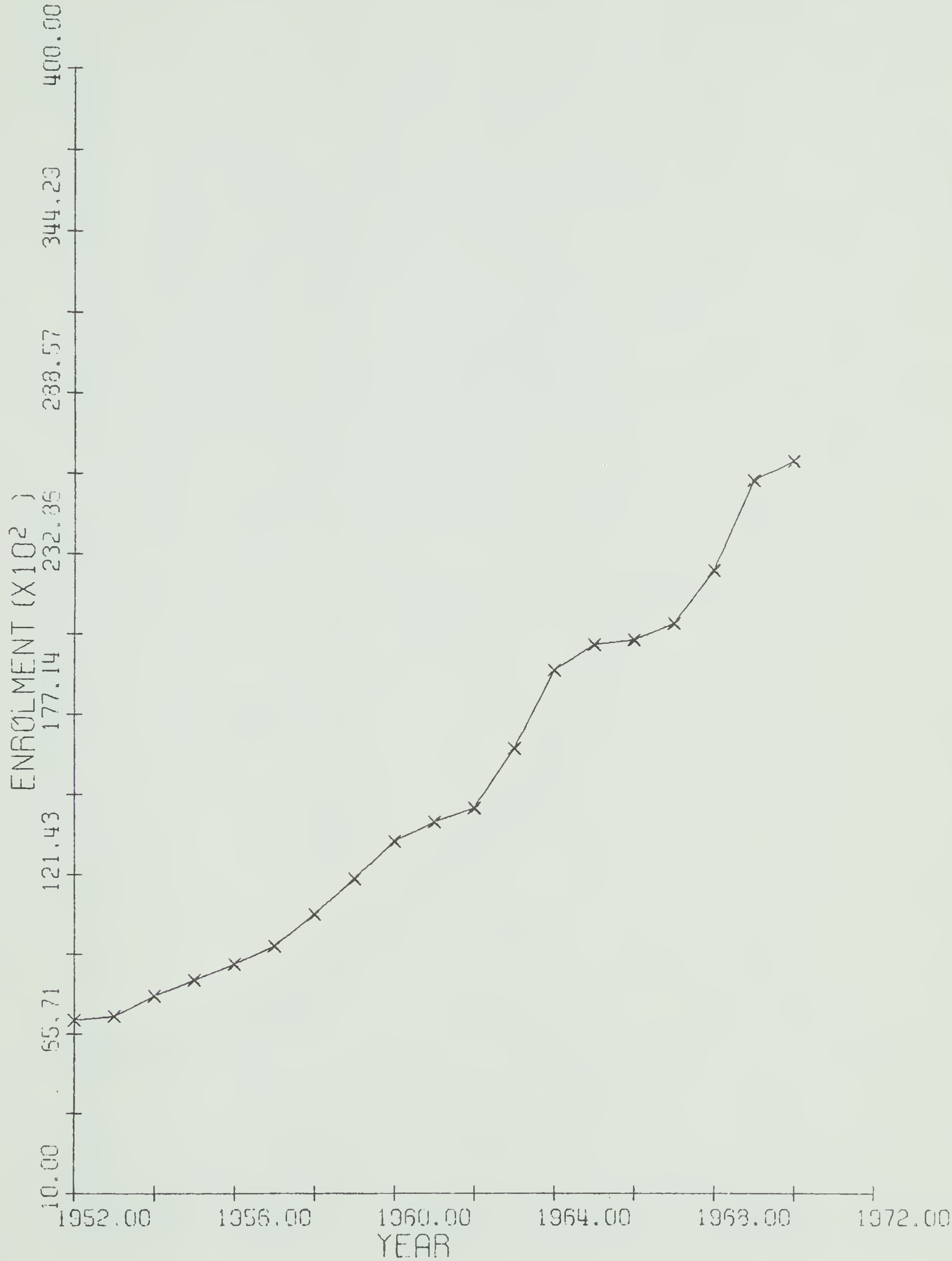

FIGURE 14

Grade XI Enrolment by Year

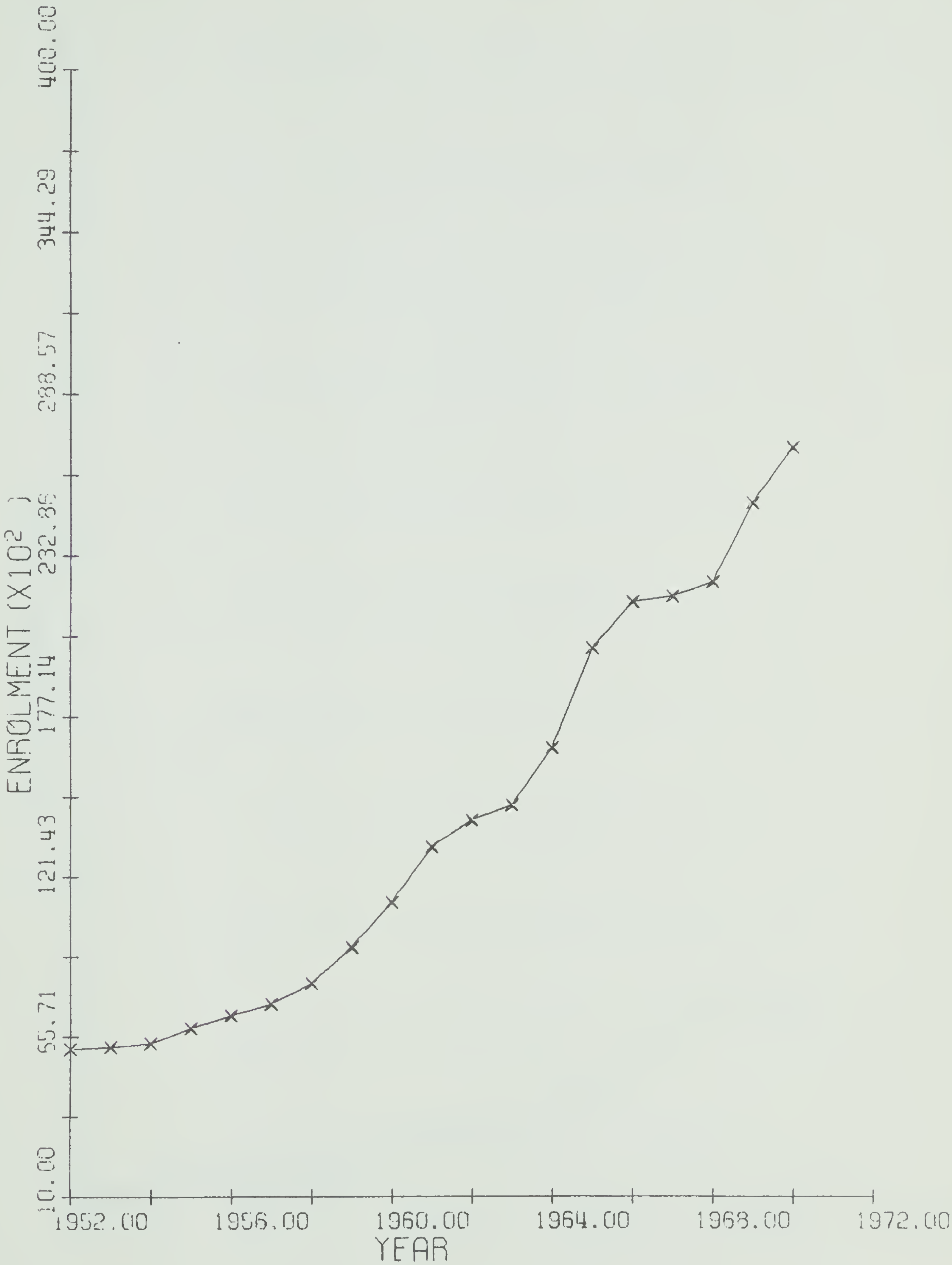

FIGURE 15

Grade XII Enrolment by Year

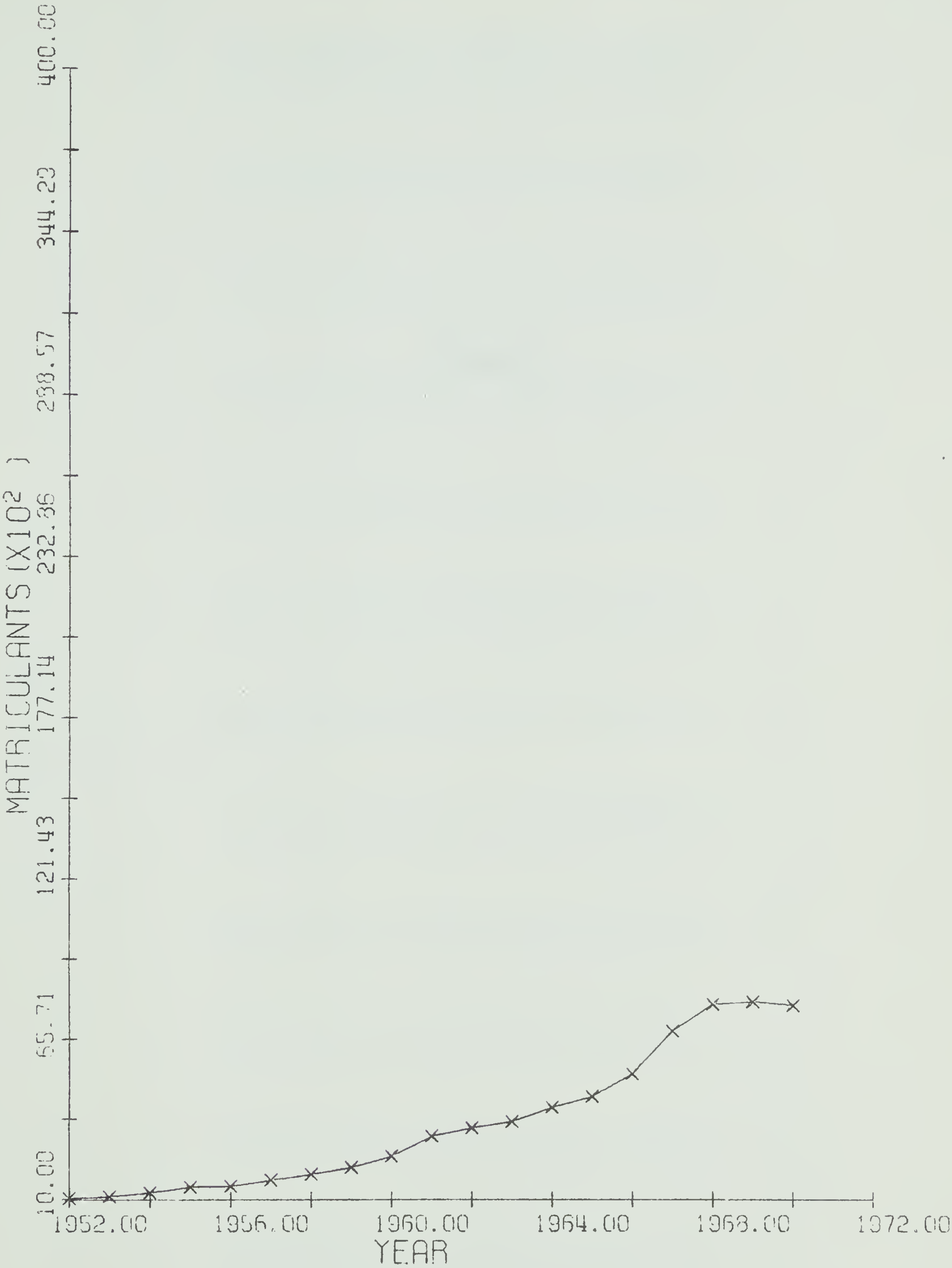

FIGURE 16

Number of Senior Matriculants by Year

APPENDIX B

TABLE IV

Actual Survival Ratios by Year

Year	Grade*											
	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Sr.Mat.**
1953	0.9143	1.0143	0.9992	0.9932	0.9945	0.9859	0.9303	0.9061	0.7843	0.7991	0.8811	0.1766
1954	0.9255	1.0090	0.9972	0.9937	1.0002	0.9945	0.9400	0.9243	0.7842	0.8200	0.8814	0.1950
1955	0.9538	0.9934	0.9899	0.9947	0.9969	0.9961	0.9445	0.9410	0.8100	0.8541	0.8712	0.2072
1956	0.9808	0.9917	0.9895	0.9953	0.9943	0.9918	0.9421	0.9484	0.8058	0.8326	0.8640	0.1992
1957	0.9645	1.0056	0.9918	0.9954	1.0051	0.9959	0.9457	0.9460	0.8125	0.8244	0.8554	0.2160
1958	0.9738	1.0026	0.9927	0.9929	0.9931	1.0018	0.9596	0.9669	0.8340	0.8508	0.8749	0.2216
1959	0.9709	1.0081	0.9913	1.0076	0.9848	1.0064	0.9636	0.9674	0.8406	0.8759	0.9020	0.2181
1960	0.9801	1.0095	1.0055	1.0058	0.9941	1.0152	0.9685	0.9792	0.8451	0.8763	0.9386	0.2216
1961	0.9763	1.0080	0.9925	1.0010	0.9927	1.0195	0.9624	0.9791	0.8401	0.8927	0.9909	0.2422
1962	0.9781	0.9810	0.9841	0.9927	0.9736	1.0135	0.9584	0.9780	0.8367	0.9012	1.0099	0.2466
1963	0.9818	0.9788	1.0079	1.0039	0.9934	1.0291	0.9684	0.9919	0.8492	0.9117	1.0128	0.2525
1964	0.9786	0.9803	0.9961	0.9972	0.9877	1.0256	0.9667	0.9915	0.8560	0.9286	1.0060	0.2523
1965	0.9839	0.9724	0.9990	0.9947	0.9876	1.0194	0.9681	0.9929	0.8735	0.9400	1.0444	0.2274
1966	0.9803	0.9651	1.0029	0.9880	0.9855	1.0183	0.9647	1.0015	0.8718	0.9212	1.0782	0.2476
1967	0.9800	0.9777	1.0097	0.9889	0.9830	1.0244	0.9531	1.0022	0.8924	0.9236	1.0783	0.3134
1968	0.9957	0.9818	1.0109	0.9818	1.0038	1.0311	0.9782	0.9824	0.9312	0.9256	1.0726	0.3478
1969	0.9947	0.9944	1.0089	0.9884	0.9951	1.0297	0.9862	0.9637	0.9684	0.9547	1.1058	0.3136
1970	0.9661	0.9766	0.9718	0.9881	0.9918	1.0178	0.9734	0.9674	0.9895	0.9325	1.0461	0.2869

* Ratio of Grade Enrolment to Enrolment in Preceding Grade, one year previous.

** Ratio of Senior Matriculants to Grade 12 Enrolment in the same year.

TABLE V

Projected Survival Ratios by Year

Year	Grade*										Sr.Mat.**
	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Sr.Mat.**

Year	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Sr.Mat.**
1953	0.9143	1.0143	0.9992	0.9932	0.9945	0.9859	0.9303	0.9061	0.7843	0.7991	0.8811	0.1760
1954	0.9267	1.0130	0.9979	0.9948	0.9942	0.9926	0.9368	0.9208	0.7955	0.8179	0.9032	0.1826
1955	0.9333	1.0107	0.9968	0.9952	0.9963	0.9972	0.9417	0.9305	0.7975	0.8294	0.9072	0.1963
1956	0.9483	1.0029	0.9933	0.9958	0.9969	1.0007	0.9467	0.9432	0.8077	0.8498	0.9037	0.2103
1957	0.9697	0.9959	0.9904	0.9964	0.9962	1.0011	0.9491	0.9548	0.8134	0.8562	0.8955	0.2123
1958	0.9794	0.9960	0.9892	0.9967	0.9959	1.0022	0.9516	0.9614	0.8194	0.8553	0.8836	0.2220
1959	0.9885	0.9958	0.9889	0.9958	0.9949	1.0047	0.9582	0.9728	0.8313	0.8622	0.8803	0.2296
1960	0.9923	0.9984	0.9885	1.0007	0.9907	1.0079	0.9645	0.9804	0.8425	0.8758	0.8881	0.2306
1961	0.9967	1.0017	0.9940	1.0040	0.9909	1.0135	0.9708	0.9891	0.8519	0.8855	0.9090	0.2319
1962	0.9968	1.0041	0.9938	1.0046	0.9908	1.0192	0.9726	0.9944	0.8559	0.8978	0.9468	0.2438
1963	0.9957	0.9955	0.9903	1.0015	0.9834	1.0209	0.9713	0.9962	0.8560	0.9093	0.9846	0.2523
1964	0.9948	0.9874	0.9966	1.0029	0.9851	1.0276	0.9732	1.0013	0.8591	0.9207	1.0145	0.2595
1965	0.9918	0.9815	0.9974	1.0013	0.9848	1.0309	0.9734	1.0038	0.8632	0.9345	1.0326	0.2623
1966	0.9905	0.9743	0.9990	0.9989	0.9848	1.0304	0.9735	1.0051	0.8721	0.9480	1.0578	0.2478
1967	0.9876	0.9661	1.0016	0.9942	0.9843	1.0286	0.9717	1.0082	0.8779	0.9495	1.0875	0.3134
1968	0.9848	0.9653	1.0062	0.9907	0.9830	1.0288	0.9654	1.0100	0.8894	0.9789	1.1075	0.3175
1969	0.9886	0.9674	1.0102	0.9854	0.9902	1.0312	0.9696	1.0029	0.9128	0.9470	1.1166	0.3382
1970	0.9914	0.9751	1.0123	0.9838	0.9931	1.0323	0.9764	0.9888	0.9454	0.9551	1.1320	0.3291

* Ratio of Grade Enrolment to Enrolment in Preceding Grade, one year previous

** Ratio of Senior Matriculants to Grade 12 Enrolment in the same year

FIGURE 17

Actual and Forecast Survival Ratios

For Grade I to Grade II by Year

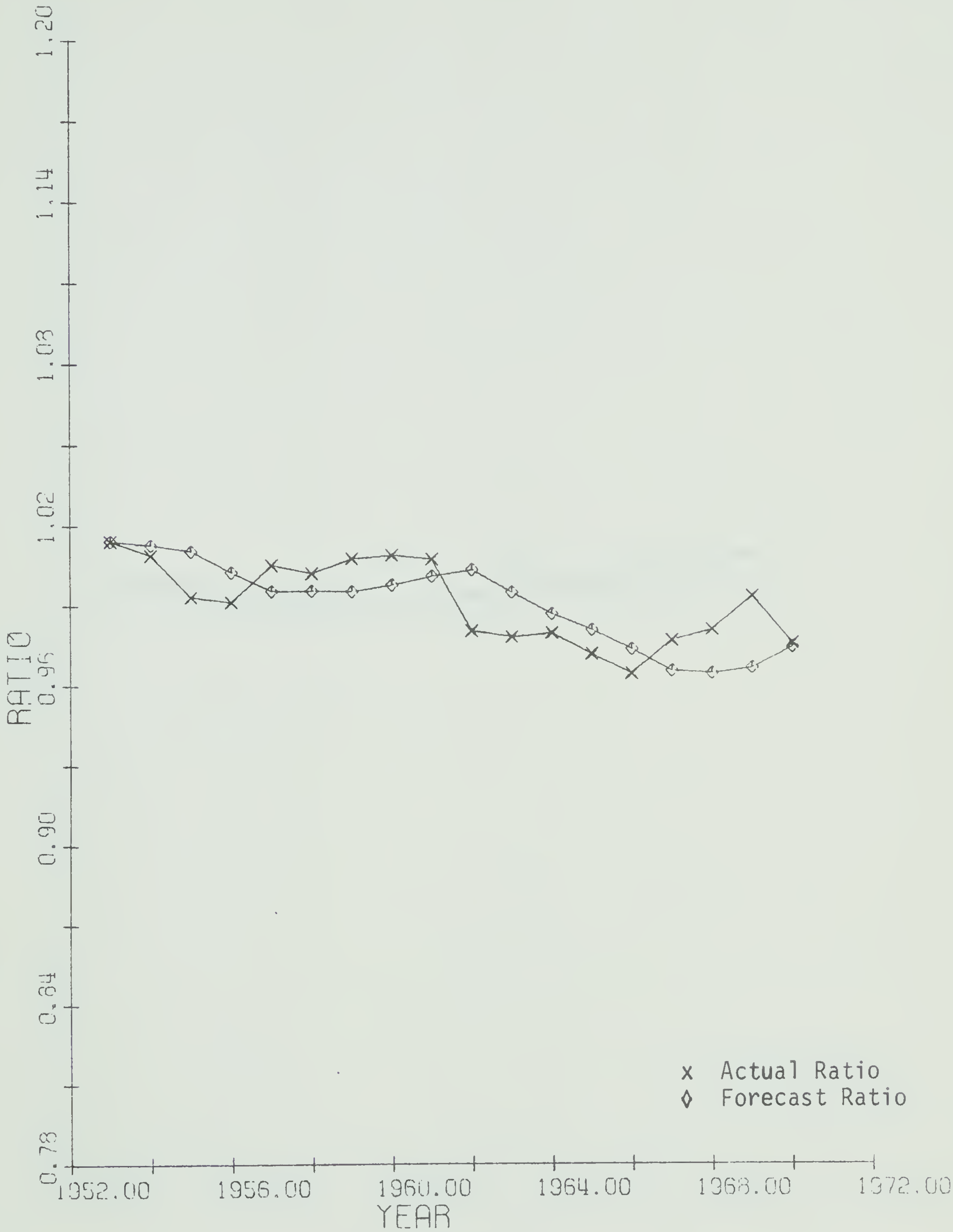

FIGURE 18

Actual and Forecast Survival Ratios

For Grade II to Grade III by Year

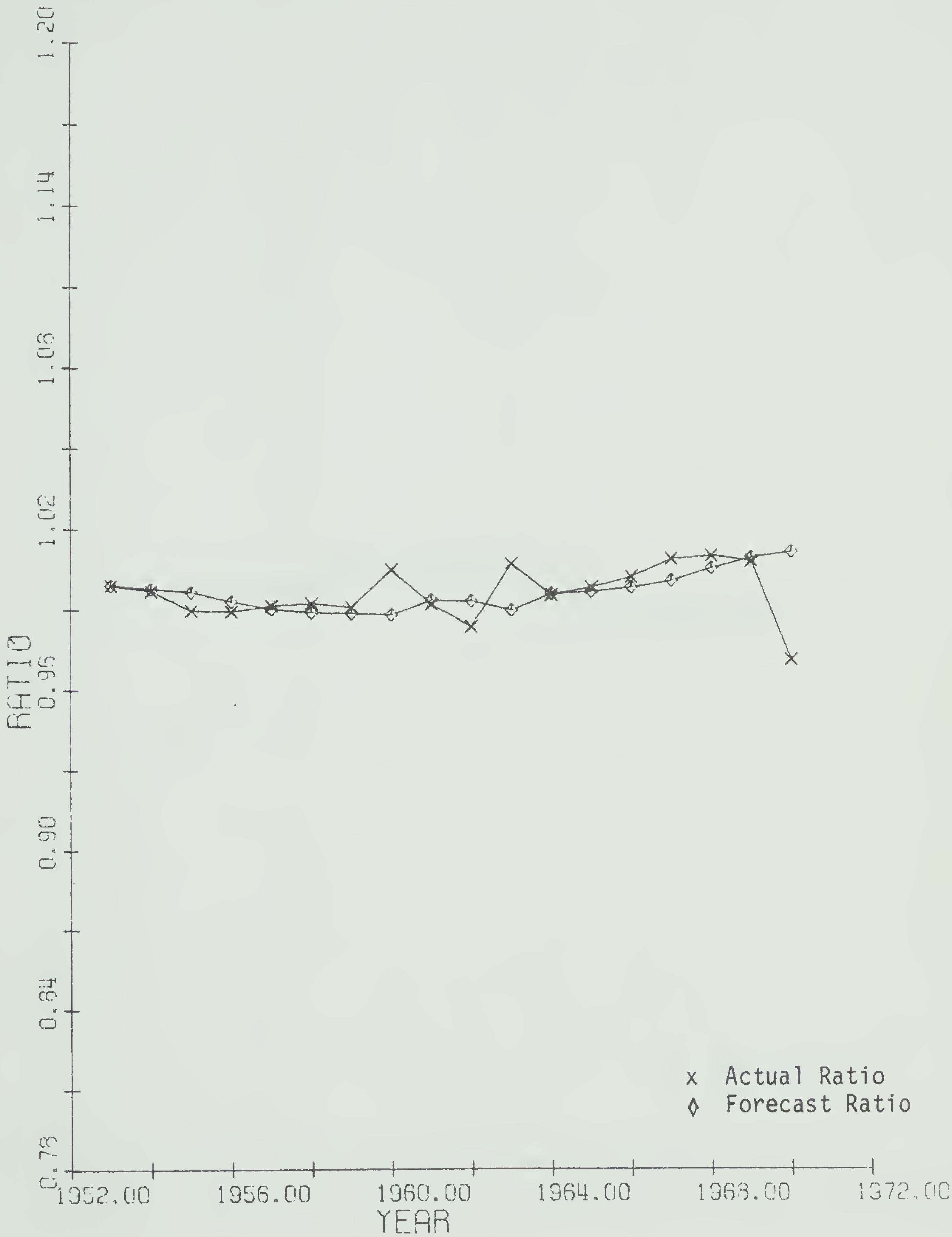

FIGURE 19

Actual and Forecast Survival Ratios

For Grade III to Grade IV by Year

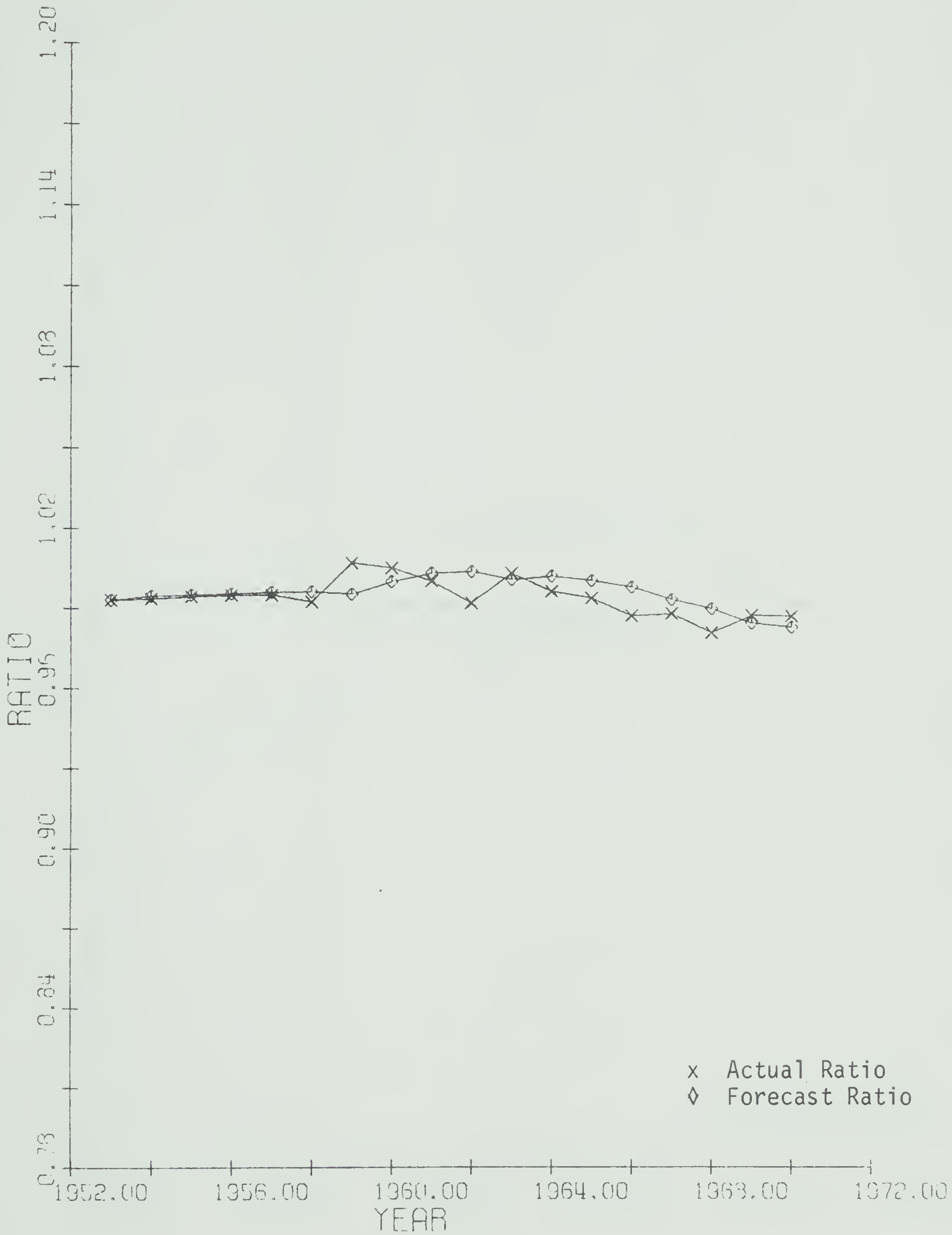

FIGURE 20

Actual and Forecast Survival Ratios

For Grade IV to Grade V by Year

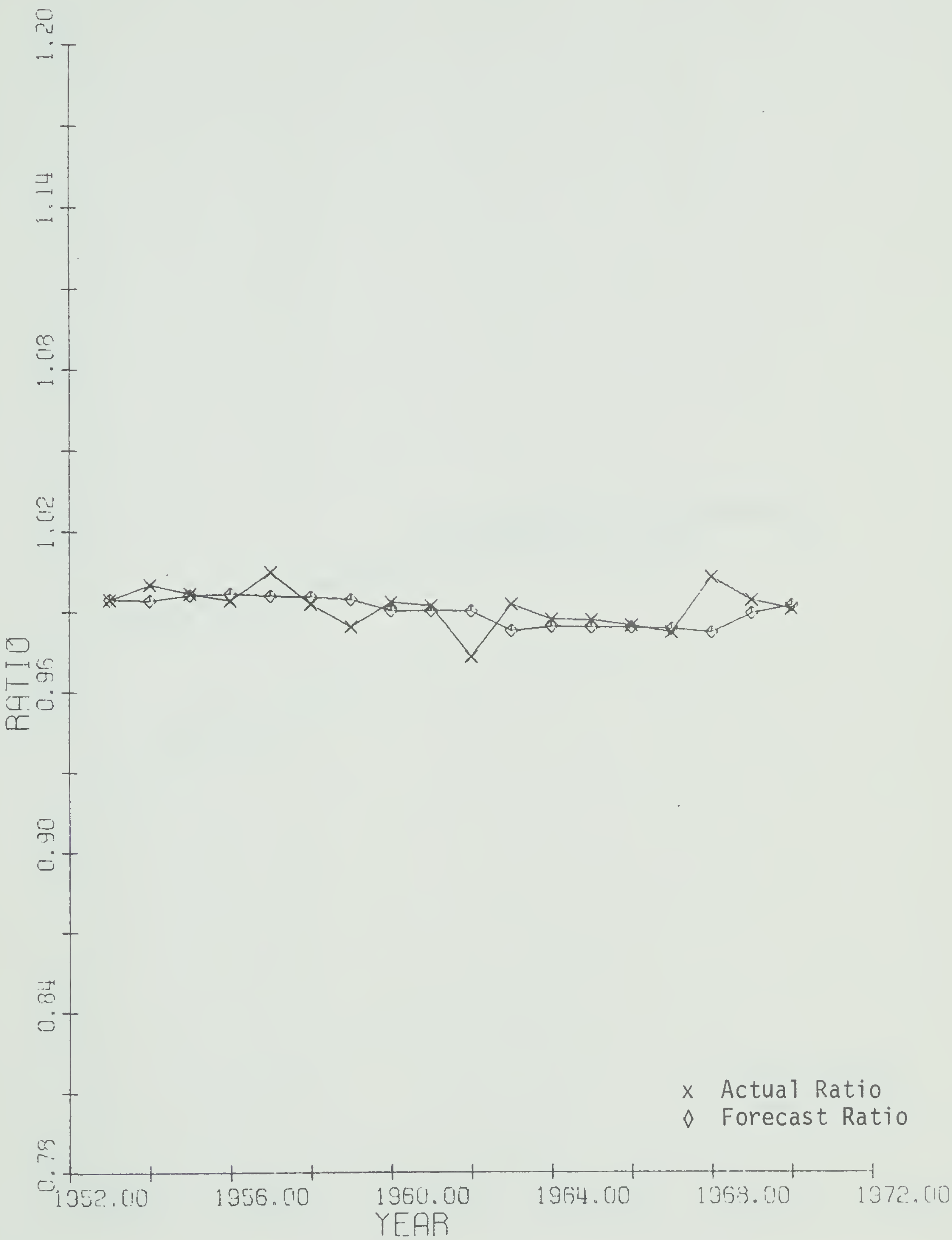

FIGURE 21

Actual and Forecast Suvival Ratios

For Grade V to Grade VI by Year

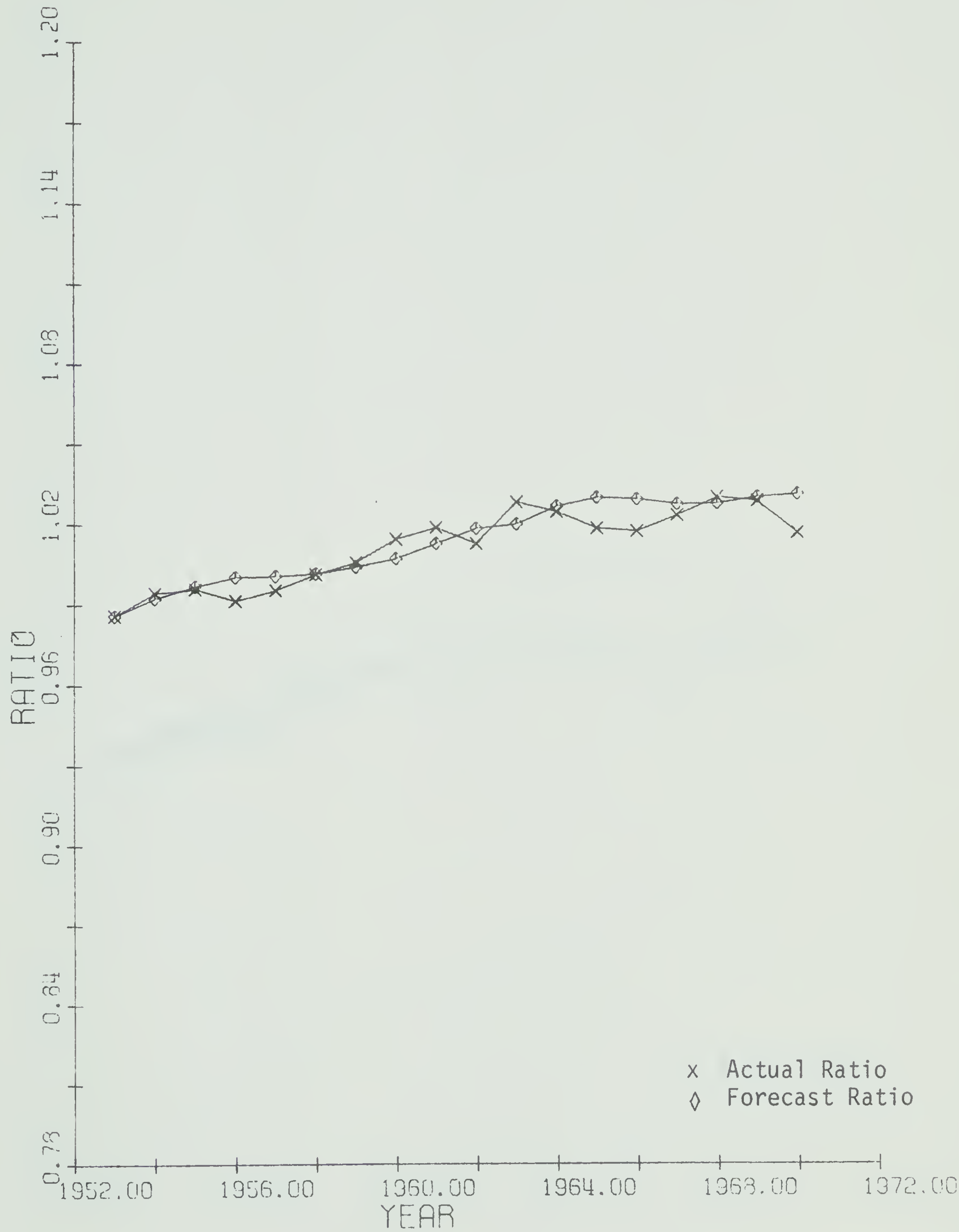

FIGURE 22

Actual and Forecast Survival Ratios

For Grade VI to Grade VII by Year

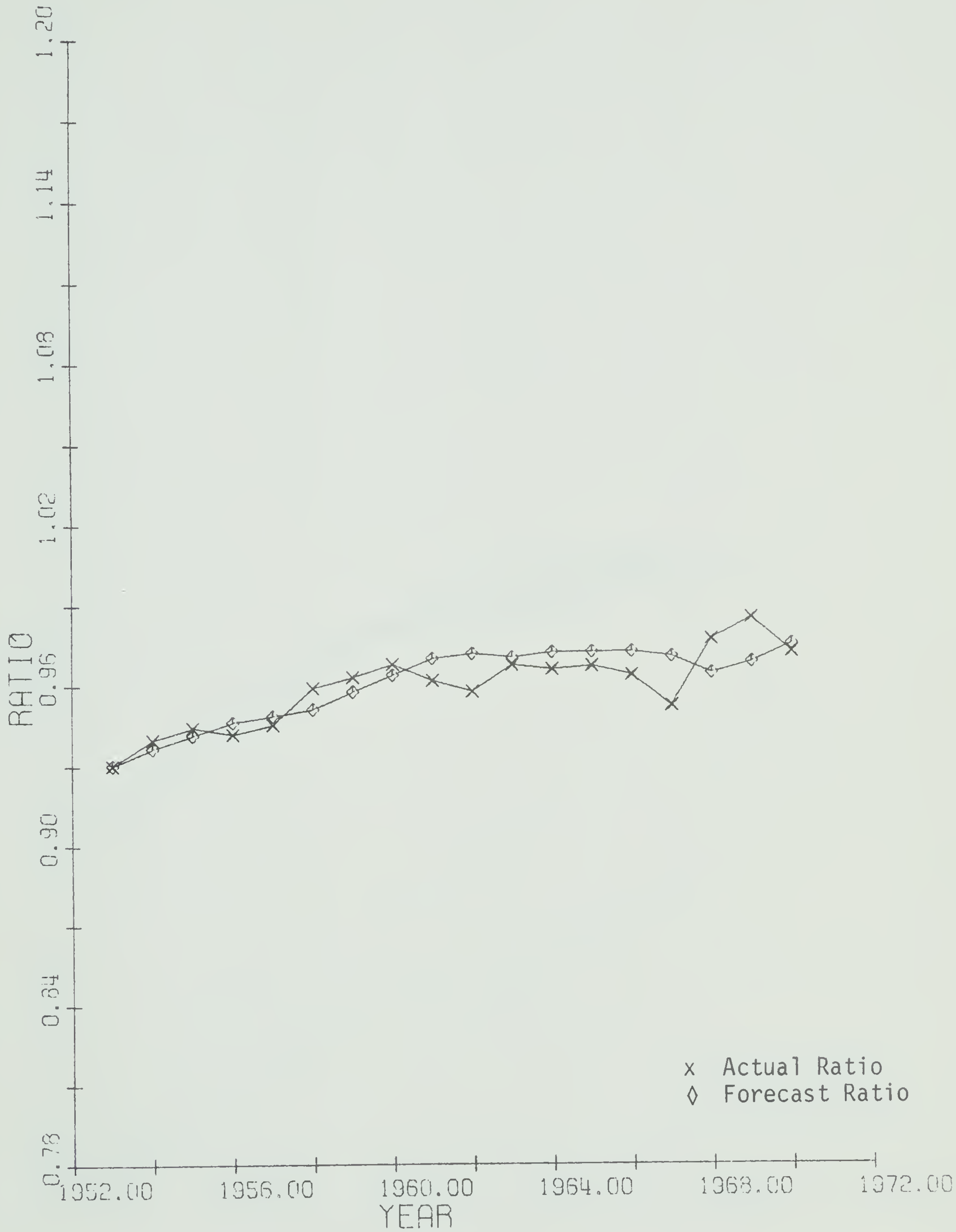

FIGURE 23

Actual and Forecast Survival Ratios

For Grade VII to Grade VIII by Year

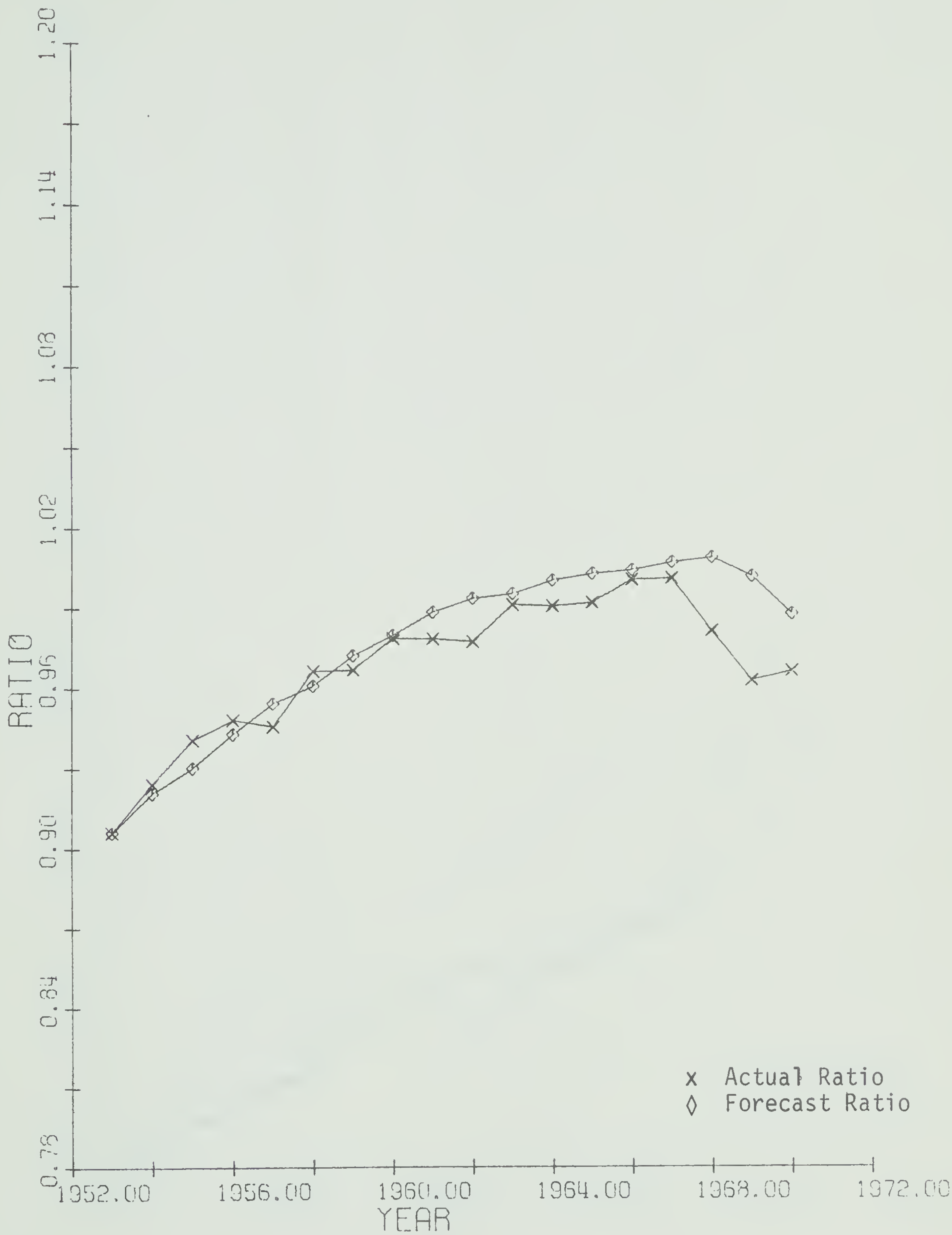

FIGURE 24

Actual and Forecast Survival Ratios

For Grade VIII to Grade IX by Year

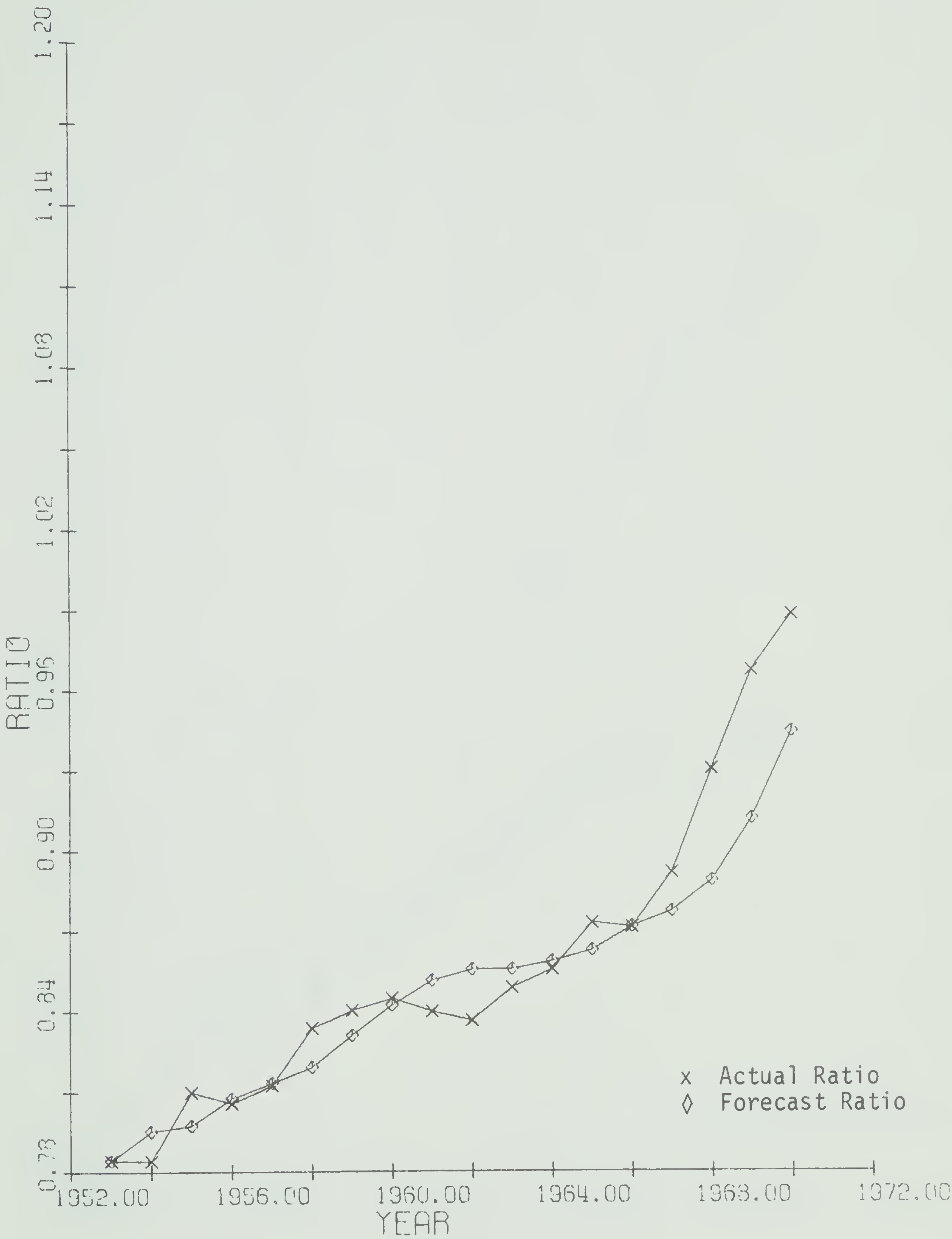

FIGURE 25

Actual and Forecast Survival Ratios

For Grade IX to Grade X by Year

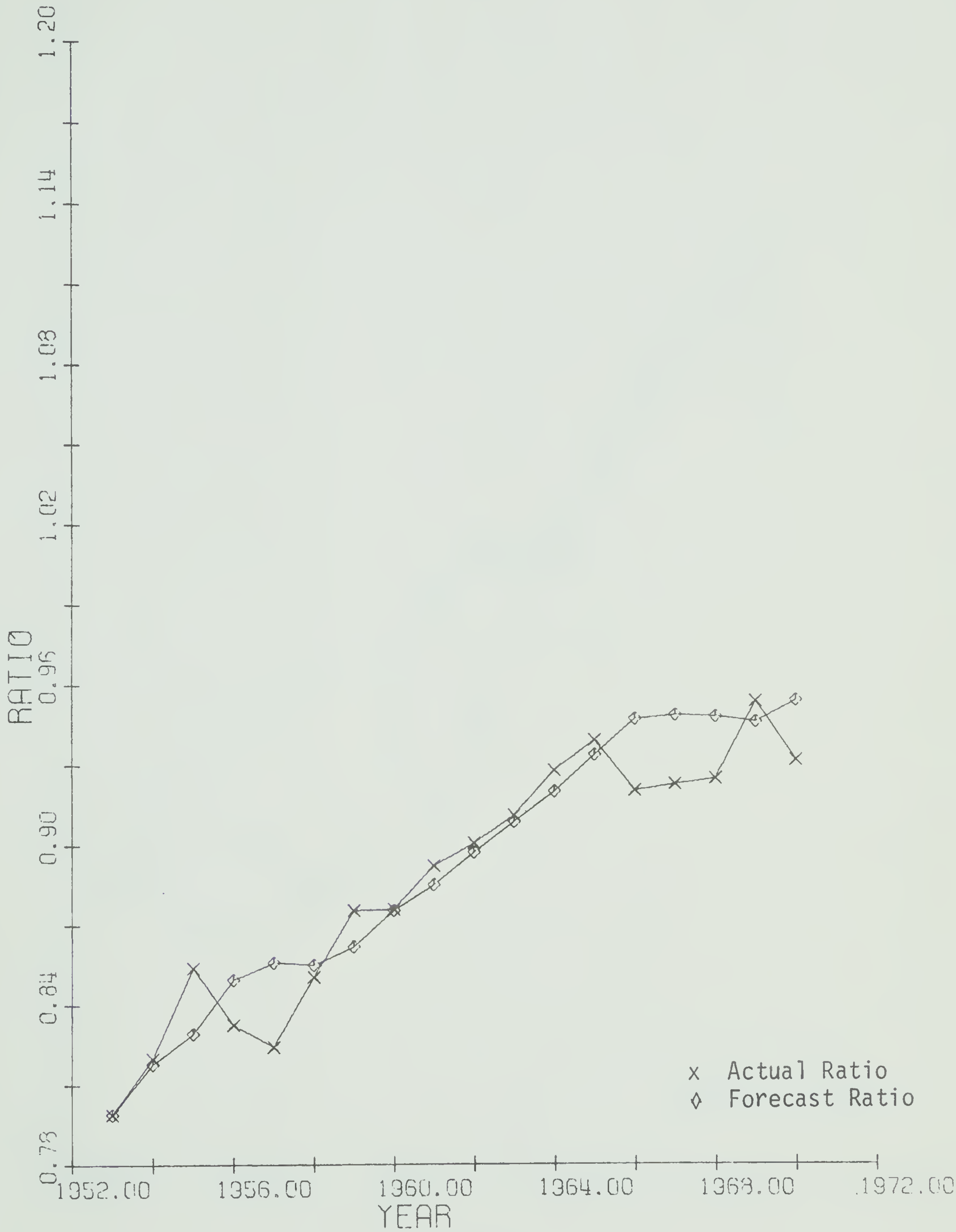

FIGURE 26

Actual and Forecast Survival Ratios

For Grade X to Grade XI by Year

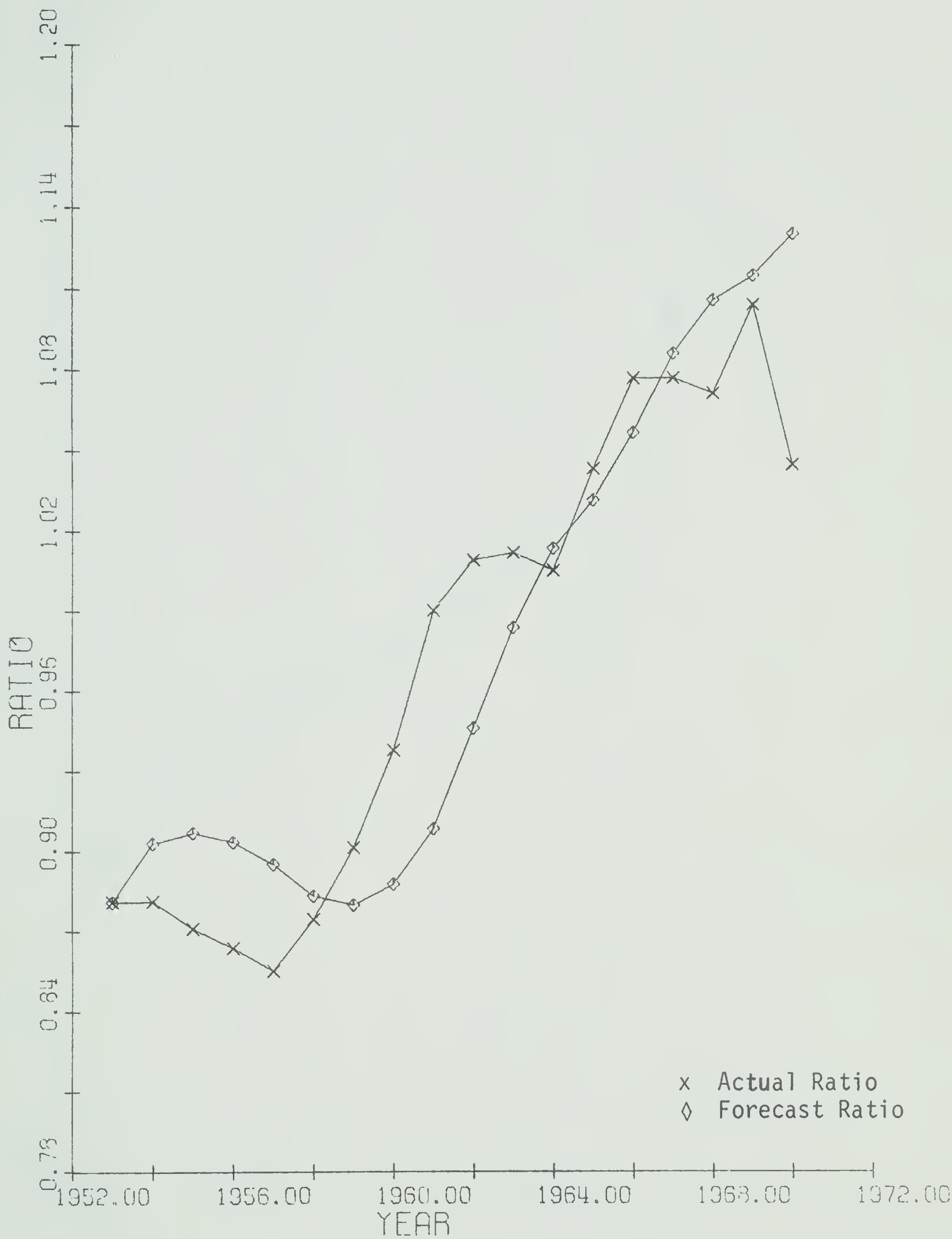

FIGURE 27

Actual and Forecast Survival Ratios

For Grade XI to Grade XII by Year

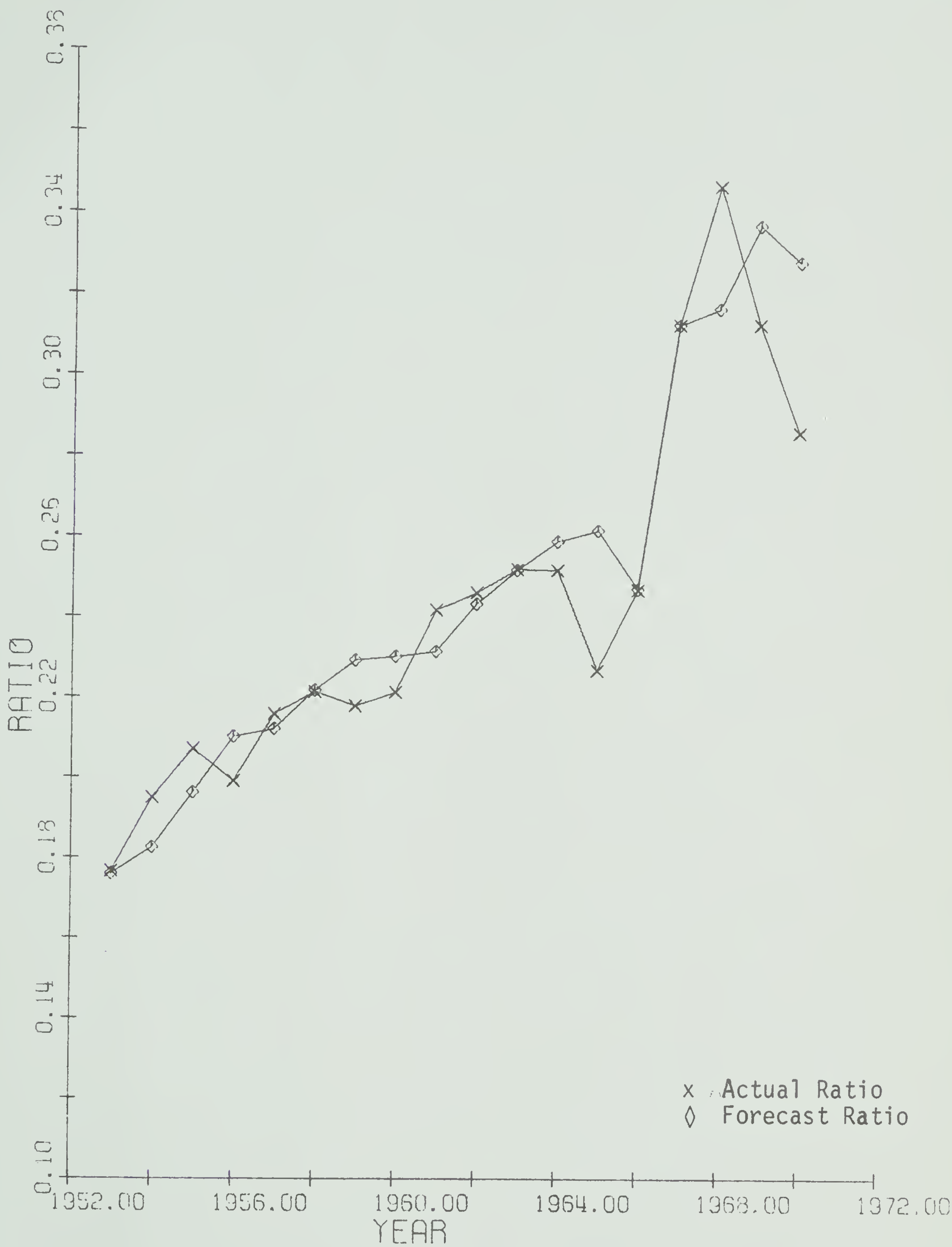

FIGURE 28

Actual and Forecast Survival Ratios

For Grade XII to Senior Matriculation by Year